

# TEACHER TELLING IN THE MATHEMATICS CLASSROOM: A MICROLEVEL STUDY OF THE DYNAMICS BETWEEN GENERAL AND CONTEXTUALIZED KNOWLEDGE

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**ABSTRACT.** In this article, we analyze moments of teacher telling (MTT) involving the exposition of new knowledge to students. We first specify the theoretical framework used for our analyses and describe our global methodology, focusing on teacher telling moments as taking part in the students' mathematics learning. Then, we review the literature on this topic and develop a specific tool, called a "proximity," to study MMTs in relation to whole-class scaffolding. Finally, we compare two high school teachers' practices in teaching the same content – variation of functions for 10th grade students – to illustrate this new analytical lens. In the conclusion, we discuss our approach and develop several research perspectives.

**LES MOMENTS D'EXPOSITION DE NOUVELLES CONNAISSANCES PAR L'ENSEIGNANT EN COURS DE MATHÉMATIQUES : UNE ÉTUDE DE MICRO-NIVEAU SUR LA DYNAMIQUE ENTRE LES CONNAISSANCES GÉNÉRALES ET CONTEXTUALISÉES**

**RÉSUMÉ.** Ici, nous présentons comment nous abordons l'étude des moments d'exposition de nouvelles connaissances aux élèves par les enseignants. Nous précisons premièrement le cadre théorique que nous utilisons pour nos analyses et explicitons notre méthodologie globale, en insistant sur le fait que les moments d'expositions de connaissances par l'enseignant participent à l'apprentissage des mathématiques par les élèves. Nous faisons ensuite une revue de la littérature sur ce sujet. Nous développons un outil spécifique, appelé « proximités », pour étudier ces moments, en relation avec un encadrement de la classe entière. Enfin, nous comparons les pratiques de deux enseignants du secondaire sur le même contenu pour illustrer cette nouvelle approche analytique. En conclusion, nous discutons de notre démarche et développons plusieurs perspectives de recherche.

In this article, we study moments of teacher telling (MMT) involving the exposition of new pieces of mathematical knowledge to students from the 10th grade.

What students do during an MMT is not easy to characterize or even observe because students are usually listening to, or taking notes on, what the teacher is presenting. Nevertheless, we believe these moments do contribute to students' learning, and this study aims to improve understanding of how MMTs might be making an impact in the learning process. Are there different ways of exposing students to mathematics, with different consequences in terms of students' mathematical activities?<sup>1</sup> For example, a teacher may choose to give examples before recounting the lecture, or to show or not show proofs of theorems. Students can be asked to read their textbook or watch a video before the MMT. Sometimes, students are left to discover or establish certain properties on their own, depending on the content and other circumstances. During the telling itself, relationships to previous knowledge or activities may be explained to a greater or lesser extent, or students may be asked to share their insights, which may be discussed based on the specific mathematical concepts being studied. However, the initial concern remains: How can researchers study MMTs and their actual impact on learning since it is difficult to observe students' activities during those moments?

In this article, we open by clarifying the issues at hand, beginning first with our theoretical hypotheses. We provide some details regarding MMTs and a glimpse into the ordinary practices of mathematics teachers in France when introducing pieces of knowledge to their students. We present our general methodology for studying students' activities. We then refer to the literature about teacher telling and whole-class scaffolding to highlight the way our point of view can shed light on these questions. In the third part, we introduce a tool, called a "proximity," leading to a methodology that may be used to study whole-class scaffolding during MMTs. In the fourth part, we illustrate our methodology and highlight our use of proximities to detect diversity in the implementation of the same mathematical content by analyzing MMTs in two classes. In the conclusion, we summarize our work and discuss the implications of our research and results.

#### **FROM STUDENTS' ACTIVITIES TO STUDENTS' LEARNING IN MATHEMATICS: OUR GENERAL FRAMEWORK AND METHODOLOGY**

The general outline of our research begins with our specific use of activity theory (as specified below), postulating that students' mathematical activities are the basis of their learning. However, these activities are not easy for the researcher to observe directly. Students' activities are mostly brought about by the teacher's choices in regard to mathematical content and its implementation in the classroom. Indeed, students' activities performed under a given piece of content depend on the specific tasks chosen by the teacher to be worked on (i.e., their order, their place in relation to the MMT, and their degree of complexity, especially

with regard to the use of pieces of knowledge requiring adaptations). Both the way tasks and activities contribute to constructing meaning and their degree of technicality are involved in our analyses. Students' activities also depend on the way in which the students work, what we call "implementation of the task": alone or in small groups, with more or less time, more or less interventions from the teacher, with direct or indirect help in the event of blockage, and with corrections that are based on the students' solutions.

The more varied the adaptations of knowledge required to solve the tasks, and the greater the teachers' interventions are adapted to the actual work of the students, the better the potential of the teaching is to make a large number of students learn, according to our hypothesis.

As such, we have to study teachers' choices to better understand the mathematical activities made possible for students in the classroom.

### *A specific use of activity theory*

Our hypothesis is that learning is triggered by activity: It stems from both the constructivist Piagetian and socioconstructivist Vygotskian approaches to learning. We rely on the complementarity of the theoretical frameworks underlying these approaches regarding conceptualization, as each of them proposes an original perspective on the construction of knowledge<sup>2</sup> (Robert, 2012; Rogalski, 2013). In fact, we transpose the theoretical concepts of Piaget and Vygotsky about child development in daily situations to the context of students learning mathematics in classroom situations.<sup>3</sup>

Vygotsky has also developed the crucial concept of the *zone of proximal development* (ZPD), which can be defined as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). In the teaching and learning context, the ZPD is the area between what a learner can do by themselves and what they can perform with the help of a more knowledgeable other, the teacher, or a peer.

The ZPD is constantly changing (although not quickly), and an appropriate teaching process consists of dialectics between scaffolding students' mathematical activity and carefully reducing support on a given piece of content. We emphasize the importance of the mediating role of the teacher (as a "better knower"); in this regard, we utilize Bruner's (1983) concepts related to mediation as a scaffolding process. More precisely, we introduce the study of the way the teacher helps the students, the comments they add, the questions and answers that occur, and their attempts to connect a new piece of knowledge with what the students already know (or can solve). Our aim is to understand the possible effects of these interventions on students' mathematical activity.

### *Some specificities of MTTs in the classrooms*

Moments of teacher telling (MTTs), when teachers expose a piece of knowledge, are crucial moments in the learning process where generalization, formalization, and organization of a new piece of knowledge – in its articulation within previous (or pre-learned) knowledge – are at stake. During the MTTs, some pieces of knowledge are not to be directly used by the students in a specific task, and the whole meaning of the discussed content might not be immediately accessible to the students. Nevertheless, we admit, on the basis of our theoretical framework, that MTTs – as long as they are not isolated from the rest of the process – play a role in students' conceptualizing process. Indeed, MTTs are part of the dialectical movement between general aspects of knowledge and contextualized ones.<sup>4</sup> They constitute the origin of the students' activities with a given concept, as well as where, when, and how a certain piece of knowledge may be used; in the long term, MTTs contribute to the construction of meaning made by the students. Their effects on this construction depend on the aimed level of conceptualization, the age of the students, and the content at stake.

The connections to be made, between contextualized and general forms of knowledge, or between new and previous pieces of knowledge, require filling some gaps that might differ among students, depending on their activities or knowledge. We assume that it is important for the teacher to identify these gaps and help fill them. Hence, closing the gaps means, among other things, clarifying as many implicit aspects as possible, according to the students' actual understanding, as expressed by at least some of them. This is one of the main issues during MTTs:<sup>5</sup> the need for teachers to introduce new knowledge, while not straying too far from what the students actually know and do.

Of course, the exposition of knowledge by the teacher constitutes only a part of the process in students' aimed learning. Sometimes, ahead of an MTT, *introductory tasks* are proposed to students. It may be a particular case where a general concept has to be used (partially), or a modeling task, or the use of specific words, first contextualized by being encountered in examples. The idea is to create, for the students, a need or a motive to learn this new piece of knowledge (based on our constructivist perspective on learning). Let us provide an example: Students in the second-to-last year of secondary school (13–14 year olds, 8th grade) are required to learn the Pythagorean theorem. The teacher could introduce it at the beginning of the chapter, and they might or might not provide a proof of it. The teacher could also tell the students before introducing it (or just after) that this theorem links the geometrical property of a triangle being right-angled with a numerical property about the lengths of the sides. However, an introductory task could also consist of spending some time before the lesson having students draw triangles, measuring the sides, and comparing two numbers: the square of the length of the longest side and the sum of the squares of the lengths of the other two sides. A discussion could then lead to a conjecture about the

triangles where the equality between these two numbers seems to be verified, thus preparing them for the lesson. The question of approximating measurements can motivate the demonstration of the Pythagorean theorem in the case of the right-angled triangle. Here, the pupils can explore the generalization of their findings. There are other types of introductory tasks, but the aim is always to prepare students for the new knowledge, through some work in a particular context or by making students feel the need for this knowledge to solve the tasks at stake. Our hypothesis is that such introductory tasks make it easier for students to make sense of the MTT (and, importantly, to listen to it).

In teaching sessions following an MTT, the students usually have to use the new general knowledge in exercises, with or without *adaptations* (i.e., variations in the way knowledge is put to work), in various contexts, with more or less time and range of initiative, depending on the content. Teachers usually expect their students to both memorize the wording of a given definition or property and to be able to use it in various exercises. Analyzing an MTT then does not only require describing the mathematical definitions and properties which are the focus of the teaching but also the various associated tasks to be solved and their implementation in class.

### *Generalities in our methodology of studying learning*

To evaluate the possible student activities generated by the teaching, and their potential in terms of conceptualization of a given new piece of knowledge, we have to take into account the specificity of each particular piece of mathematical knowledge. We have thus introduced the idea of “highlights” of a mathematical concept, which refers to the intersection of a mathematical point of view, the school curricula, and the cognitive difficulties for students on this subject. It offers the researcher a global reference point to study the content at stake in the teaching and learning process and to compare what may be potentially expected (by the researcher) and what effectively occurs in class. It acts as a global a priori analysis of the content to be taught (Bridoux et al., 2016, p. 191), and is critical to our analyses.

However, analyzing students’ activities leads to difficulties for the researcher from a methodological point of view. First, activities related to a targeted mathematical concept should be considered in their entirety over the long term to understand what truly happens in this learning process. Furthermore, following the work of Leontiev (1972/1978), we distinguish activities from tasks and do not identify activities with observable actions only, as activities have a mental component that remains inaccessible to the researcher. However, we assume that we can reasonably infer the possible activities of the students from both the proposed mathematical tasks and the choices made for their implementation in the classroom. When there are no obvious tasks, as is the case during the MTTs, we have to adapt our methodology in another way, as we explain below.

## LITERATURE ABOUT LECTURES IN MATHEMATICS CLASSROOMS

*International context*

Since the 1980s, many countries, including France, have initiated reforms in mathematics education. With a more constructivist view of the basics of learning, these reforms tend to reconsider the traditional ways of teaching mathematics, questioning the model of teaching as information transmission (the “telling model”; Smith, 1996, p. 393). Regardless of their initial motives, they have widely promoted “an active view of learning mathematics” (Smith, 1996, p. 393) in which the role of the teacher is fundamentally different from their role in a more traditional model. In particular, “teachers must play more the role of classroom facilitator than knowledge source” (Smith, 1996, p. 394). However, MMTs have not disappeared from classrooms. One of the reasons pointed out by Smith (1996) is that telling plays an important role in mathematics teachers’ sense of efficacy and that changing this and replacing it with new “moorings for efficacy” (p. 395) is not that simple. Another reason might be that even in a teaching model based on a constructivist conception of learning, teacher telling still has a role to play. Indeed, one cannot expect students to come up with usual mathematical theorems, definitions, formulas, or vocabulary without any help. This idea led Smith to come up with the notion of “judicious telling” (p. 397) as a central component of teaching practices, consistent with an active learning model. He describes the function of judicious telling as the mediation teachers have to achieve between the accepted mathematical knowledge and methods and the particular intellectual communities of their classrooms. In particular, teachers are in charge of “additions – such as useful terminology, ways of representing mathematical ideas and counterexamples to student conjectures – in settings where those additions are necessary” (Smith, 1996, p. 397).

The French word “*cours*,” referring both to the whole session, to the part of the session devoted to the exposition of pieces of knowledge by the teacher, and to the resulting written record, shows the place teacher telling has been taking in French culture. Indeed, before the constructivist views on education led to more involvement for students in their learning, mathematics sessions were mainly in the form of lectures from teachers. Today, in many mathematics classes in secondary school throughout France, and for a significant proportion of sessions, a large part is still devoted to exposition by the teacher; this is done, partly in writing and partly orally, for certain general mathematical objects (theorems, properties, formulas, and definitions, as well as vocabulary, methods, and proofs) and comments about these objects.

*The relations between MTT analysis and the mathematical content at stake*

Constructivist approaches in teaching mathematics led to reforms emphasizing the importance of students autonomously searching for meaningful mathematics problems, with collective discussions in the classroom. Institutional reforms

in various countries (National Council of Teachers of Mathematics [NCTM], for instance) were sometimes interpreted as a requirement for teachers to listen to the students' proposals and discussions while avoiding the provision of substantive mathematical help through indicating the relevant concepts or procedures (Simon, 2013). Stein et al. (2008) discussed this interpretation of problem-oriented teaching and learning and stressed the need for teachers to be helped in moving beyond show and tell, and to maintain a sense of efficacy that they might fear losing (as Smith, 1996, had already analyzed), by giving them other means of presenting mathematical knowledge through telling while remaining in control of the mathematical dynamics and teaching goals. For further learning of mathematics in secondary education, Lobato et al. (2005) underlined that telling remains important because students cannot reinvent the mathematical content at stake.

Authors such as Baxter and Williams (2010) highlighted the tension – which they call the “dilemma of telling” (p. 8) – between engaging students in autonomous tasks, which could be effective in making sense of mathematical concepts, and guiding them toward meaningful learning from a disciplinary point of view. After a constructivist stance which led to some reduction of telling-focused teaching methods, there was a move toward a different instructional stance in mathematics education which opened a new space for teacher telling, this time based on scaffolding (Wood et al., 1976). In an introduction to a *ZDM – Mathematics Education* special issue on scaffolding and dialogic teaching, Bakker et al. (2015) conducted a review of the literature on teacher telling and scaffolding in which they defended Baxter and Williams' point of view that “telling is not necessarily at odds with the idea of scaffolding as long as it is contingent to the situation” (p. 1056).

Furthermore, the issue of making links between general and contextualized knowledge in a lesson is also found in the particular case of examples, about which Bills et al. (2006), at the 30th Conference of the International Group for the Psychology of Mathematics Education (PME 30), presented a survey of literature illustrating the diversity of approaches in examples used for the learning and teaching processes. However, as we will show, the dialectic between general and contextualized knowledge is not limited to the use of examples in mathematics learning and teaching.

### *Whole-class scaffolding and MTTs*

*Scaffolding* was first introduced by Bruner (1983) to operationalize the concept of ZPD. It “consists essentially of the [adult] ‘controlling’ those elements of the task that are initially beyond the learner’s capacity, thus permitting him to concentrate on and complete only those elements that are within his range of competence” (Wood et al., 1976, p. 90). Originally, scaffolding was oriented toward one-to-one interactions with a child or pupil. Since then, the range of situations in which scaffolding has been employed has expanded in several directions, three of which

we will highlight here. First, scaffolding may be used as a means of supporting a particular student's learning in a school context while appearing to be quite relevant to analyze their acquisitions (e.g., in reading or in mathematics<sup>6</sup>). Second, it can be applied to whole-class situations: Smit et al. (2013) proposed a conceptualization "that keeps as close as possible to the spirit of [its] origin, but that leaves room for features not salient in one-to-one interaction" (p. 818). Third, beyond problem solving, scaffolding may help students' understanding by relating a new piece of knowledge to an already familiar one or by better integrating that knowledge into a net of already organized concepts.

Given our own research interests, we are primarily concerned with the generalization of individual scaffolding to situations where there are more collective interactions and comments on mathematical content than one-to-one interactions. To this end, we have designed a specific tool to analyze teachers' interventions involving knowledge, called a "discursive proximity," which is particularly suitable for at least partially studying whole-class scaffolding during MTTs.

#### A TOOL TO STUDY WHOLE-CLASS SCAFFOLDING, ESPECIALLY DURING MTTs: PROXIMITIES IN THE TEACHER'S DISCOURSE

##### *Teachers' practices are a key element of our MTT analysis*

In MTTs, very few traces of students' activities are evident (compared to other moments in class): The students' tasks are not obvious, and observation of their activities is more difficult than during problem-solving moments, which has led us to adapt our existing methodology for this article's particular analysis (Horoks & Robert, 2007). Furthermore, the study of these moments concerns a small part of the teaching / learning process, which is not easy to interpret as an isolated element. We therefore use a detour to analyze an MTT's potential impact on students' learning through the study of the teacher's discourse during these moments, taking into account the students' activities before this moment, and according to the content at stake, the school level, and the students' known difficulties.

We will not provide here an exhaustive list of the various choices that a teacher may adopt for the MTT's content, the moment's level of generality or rigour, the adopted formalism, the proofs given, what still remains hidden (implicit), and so on. The organization of the MTT may also be very different among teachers: from a lecture, where the students take notes, to an interactive talk, where the students may take part. Sometimes, a teacher might only offer a solved exercise or a generic example (Mason & Pimm, 1984), instead of a general statement to be used as a general rule. In other circumstances, a teacher might add some historical context or emphasize the way the concepts are to be used.



In fact, the main entry point for the present analysis is tied to the comments the teacher adds, whether they were anticipated or improvised, when – and only when – they are directly linked to the students’ (possible) activities or (supposed) knowledge. Our focus is on the way the teacher’s discourse could contribute to the students’ understanding of the mathematical content at stake through the connections that the teacher might establish with elements that some of the students are already supposed to have absorbed. As explained above, it might be previous activities, pieces of previously acquired knowledge, or what the teacher imagines the students’ knowledge to be. This involves both the form and substance of teachers’ discourse.

This means that after studying the highlights of a mathematical concept – to pinpoint what students may know, what they might miss, or what may be difficult for them – we then study to what extent the students’ ZPD may be involved in the teacher’s comments during MTTs. We label the corresponding excerpts of the teacher’s discourse as proximities.<sup>7</sup> MTTs are related to whole-class scaffolding, but specified to telling moments and closely related to mathematical content and students’ activities. Three types of proximities will be identified in the teacher’s discourse according to their place between general and contextualized content. As researchers, we look for opportunities in which proximities could have been introduced by the teacher, deduced from the general characteristics of the mathematical content. We then study the teacher’s discourse to deduce the actual proximities offered to the students. This study is based on years of research on teaching practices and on our own experience in teacher education.

### *Studying proximities during MTTs: A specific methodology*

This section will detail our methodology in studying and categorizing proximities in the teacher’s discourse<sup>8</sup> during MTTs in class. We analyzed both the teacher’s and students’ discourse, to the extent that we could, through a video of the session. Proximities may happen through foreseen or improvised comments. We also intended to identify other comments, mathematical or meta-mathematical,<sup>9</sup> that may reach the students’ ZPD: proximities between the planned tasks and the text of the lesson, between the student’s effective activities on the previous tasks and the targeted knowledge, or between questions and answers occurring during the lesson. We then attempted to identify some missed opportunities, which are often tied to elements or links that remain implicit (according to the researcher’s point of view), possibly due to a lack of awareness on the part of the teacher, a misreading of students’ difficulties, or even a lack of time.

We were particularly interested in comments that may have reached the students’ ZPDs, or at least the ZPDs of the majority of the students in the class. These discursive proximities could contain explanations or clarifications on the meaning of the mathematical concept at hand, and about how to apply it; they could explicitly connect the activities of the students to the pieces of general knowledge behind them (“ascending proximities”), or link pieces of general knowledge to

the activities (“descending proximities”). These proximities could also concern<sup>10</sup> general or applied knowledge (“horizontal proximities”). This categorization of the comments, according to their place between general knowledge and its contextualized form, is based on the way the teacher attempts to bridge the gaps between the targeted pieces of general knowledge and the students’ detected knowledge or activities (Chappet-Paries et al., 2017b). We will provide examples of these categories in the following section. Our Vygotskian hypothesis is that if some comments connect the students’ previous knowledge and activities to the targeted new knowledge (or methods), they could be relevant for the learning expected, since they may reach the students’ ZPD.

#### A CASE STUDY: TWO EXAMPLES OF MTTs ON FUNCTION VARIATIONS FOR 10TH GRADE STUDENTS

In this section, we describe what happened in two different 10th grade classrooms during a session about the direction of variations of functions (one of these analyses is presented in detail in Chappet-Paries et al., 2017a) and demonstrate how proximities can be a tool to compare the practices of two teachers – in this case, “GE” and “MM” – implementing an MTT on the same mathematical content. The two classes as taught were not similar, but both were held in neither notably privileged nor notably disadvantaged social environments. We do not present the complete data here, which include other moments of the sessions that we have analyzed elsewhere (Chappet-Paries et al., 2017a; Robert & Rogalski, 2020), since our aim is strictly to illustrate the way we detect the proximities in the teacher’s discourse during MTTs, according to our previous analysis of the mathematical content.

##### *Highlights of the mathematics at stake*

Functions are first encountered in the last level of lower secondary school (“collège” in France, 14–15-year-old students) and then studied more thoroughly in the first year of upper secondary school (“lycée,” 15–16-year-old students). Formal definitions of increasing and decreasing functions are introduced for the first time in lycée, as well as the concept of variations (and of a variation table). These students also have to know how to study particular functions: linear functions (already partially met at the end of collège), polynomial functions, and so forth. Linear functions are the only type of function explicitly studied during the last year of collège, and is described by the formula  $f(x) = ax + b$  and associated with straight lines. Then, in lycée, students learn the following properties: “if  $a > 0$ , the function is increasing” and “if  $a < 0$ , the function is decreasing.” More precisely, during the first year of lycée, studying a function’s variations includes both the graphic and the algebraic general definitions of increasing and decreasing functions and their use in some exercises. The notion of derivatives is approached only during the following year.

During their years in collège, and even before then, the students are accustomed to interpreting graphics in terms of variations, including examples outside mathematics: For instance, they can read on a graph that a quantity on the y-axis increases. However, the connection between graphs and functions is not explicitly addressed before the last year of collège. Nonetheless, for us, that connection plays a key role in the conceptualizing of functions, and it represents a necessary and difficult transfer of what students know on graphs, toward the represented functions, and especially toward the algebraic aspects, that did not appear when working on graphs before that point (cf. Chappet-Paries et al., 2017a). For students, adopting this algebraic perspective involves the difficult connection between something global and perceptible on the one hand (the graph, where points are implicitly involved) and a relation between two pieces of information on the other hand, namely, the covariation of the coordinates of the points on the corresponding graph. It also involves a complex wording for the definition of an increasing function  $f$ , with a quantifier (explicit or not): “For any couple of numbers  $a$  and  $b$  in a given interval, if  $a < b$  then  $f(a) < f(b)$ .”

From our perspective, considering the expected level of conceptualization of functions’ variations in 10th grade in France, knowing the concept of an increasing function includes knowing all of its aspects (graph, values, and algebraic formula), and means that students can link these aspects and choose the most relevant one to solve a problem. This understanding of the concept could not come without the work done on exercises involving it. However, thanks to MTTs, students might already gather, ahead of this work, an operative idea of what is at stake. They can then recognize targeted pieces of knowledge, through the teachers’ comments, linking their activities and what was presented during the MTT.

Whatever the introductory task is before the MTT, the complete formalization of “for all  $a$  and  $b$  in a given interval, if  $a < b$  then  $f(a) < f(b)$ ” cannot be directly deduced by students from solving the task. For us, the gap is too large between, on the one hand, the global perception of the graph going upward, with readable consequences on the covariation of the abscissa and ordinate of the points of the graph, and on the other hand, the formal translation of it, with an implication involving a universal quantifier and two values of  $x$ . This translation cannot be provoked by a question before students know about the definition, which contradicts the introductory aim of an introductory task.

### *Studying MTTs in two classes*

Our methodology allows us to see differences in the two classes under analysis that might have impacted students’ conceptualization of the mathematics in question. Both teachers let students work on an introductory task before the MTT (the day or the morning before), but the tasks were not the same. Moreover, the time devoted to the introductory task, and to the presentation of (similar) content in the first MTT that followed it, differed widely between the two

classes. Finally, the main proximities we identified from the two teachers were very different. We detail these differences below.

### *The two introductory tasks*

In the first class, students worked on two given graphs, which were not explicitly associated with functions, and critically without any algebraic expression of the functions. The  $x$ -axis was the axis of time ( $t$ ), even if it was not explicitly indicated, and the  $y$ -axes both represented some physical quantity (temperature or altitude). The tasks then consisted of recognizing, describing, and interpreting the graphs (“the graph goes up on such period”) to conclude that “the temperature is rising.” Some intervals were introduced to characterize the values associated with growth, or some values to characterize the extremum, but the students only had to read them on the graphs. There were no questions leading to an algebraic interpretation of the increase since the function was not explicitly expressed in its algebraic form.

In the second class, the students worked on optimizing the area of an agricultural field, which could be modeled geometrically, leading to a quadratic function. Two variables were to be explicitly involved, one of them being a length ( $x$ ) and the other the area ( $y$ ) to be maximized. The values and the graph could be obtained with the use of appropriate software. Some questions involving the description and interpretation of the values and the graph did arise, but the questions also offered opportunities to go a little further toward the algebraic formalization, since the relation between  $x$  and  $y$  was explicit in this case as a result of the inevitable approximation of numerical and graphical approaches.

### *The two MTTs*

In both classes, we analyzed a transcript of a video which captured the MTT.<sup>11</sup>

#### 1) In GE’s classroom

We analyzed the first 10 minutes of teacher GE’s MTT in which he provided the definition of an increasing function after the prior work on the introductory task. He began by recalling the task as the description of the evolution of two phenomena called “functions.” However, he never mentioned the variable for time ( $t$ ) in this introduction. Then, he announced to the students that the goal of the lesson was to generalize what they had done during the introductory task. This comment may constitute a horizontal proximity between the aim of the task and the aim of the MTT. At the same time, the teacher projected on the blackboard the part of the textbook related to this lesson and let the students copy what was on the board if they so chose.

Then, the teacher defined an increasing function on an interval and wrote it on the blackboard: “If  $x$  increases, then  $f(x)$  also.” This definition is not yet the algebraic expression, but it does include the relation between the evolution of the values of  $x$  and those of  $f(x)$ . In addition, he added: “What does it mean

on a graph? It means that the graph is going up, as you saw in the introductory task.” Here, we see that the teacher displayed a descendant proximity instead of an ascendant one as we would have expected. If the teacher had chosen to begin with the example of the particular graph on which the students had worked, and had questioned them on a way to translate the behaviour of the function observed on the graph into words by describing the evolution of the coordinates’ values, we would have identified an ascendant proximity, even if only a few students had answered this question. However, since the teacher presented the work on the previous task as an illustration of the new definition, instead of presenting the definition as a precise generalization of what the students had done, we consider it a descendant one.

Next, the teacher added the algebraic definition, without any comment other than “It means also ...” He did not justify its difference with the previous expressions, nor its potential usefulness. He only wrote, “If  $a < b$  then  $f(a) < f(b)$ ,” without any reference, even orally, to the fact that it must be proven for any  $a$  and  $b$ , despite the complete expression being included in the students’ textbook and projected on the blackboard. The teacher also did not show on the graph the representation of the (incomplete) formula he wrote on the blackboard, which he could have done by placing, for example,  $a$ ,  $b$ ,  $f(a)$ , and  $f(b)$  for some values of  $a$  and  $b$ . Does it reveal that the teacher did not perceive the students’ likely difficulties with this expression? Or does it reveal that the teacher knew that this expression would not be used later to solve exercises, and thus pointless to spend time on at that juncture, as evidenced by the fact that the meaning of the expression was still inaccessible without solving a few exercises?

At this point of the MTT, a student posed a question to the teacher, asking, “Shall we be assessed on that?” One could interpret this question as a student’s unease with such a mysterious expression, without any link to the previous interpretations or translations of the growth of a function. Then, after having repeated the three ways of considering an increasing function (with the order of the values, on the graph and algebraically), the teacher went on to the decreasing case, with the same steps (this time including the graph interpretation of the algebraic expression for two given values  $a$  and  $b$ ).

## 2) In MM’s classroom

It took teacher MM 50 minutes to define an increasing function. First, he recalled the work of the introductory task – the algebraic expression of the function  $f$  with its interval of definition is reobtained after 14 minutes. As the students had to find the function’s maximum, the teacher first showed values of  $x$  and  $f(x)$  by means of a spreadsheet and let students describe what happens in terms of variations. Then, the same task was solved graphically by reading the graph on a computer. MM insisted explicitly on the link between the two semiotic registers of representation (table of values, graph): first growing, then reaching a maximum, and then decreasing. He also insisted on the approximation, cited

by some students, of these two approaches. In this approach, we see proximities based on the students' work, which may help them to recognize the relations between registers regarding some aspects of functions. Then, after half an hour, the teacher announced that he would begin a more general discussion of the variations of functions (a horizontal proximity). He explained first the need for appropriate words and the students proposed some options, such as "ascending" and "increasing." The teacher finally kept, as mathematicians do, the word "increasing" for a function whose graph goes up on an interval, thus constituting a first definition. Here, we have the first ascendant proximity, as the formulation given originates from the activity. Then, showing the graph and the values, the teacher went back to the task and repeated that the function is increasing on the first interval (thus now demonstrating a descendant proximity).

After this, he asked the students to find an algebraic translation of the first definition. He insisted on the importance of a rigorous translation: "Later you will be asked to anticipate the maxima of a function, without being given a graph [nor] a table, which are approximations, but through translating it in an algebraic language." There is a need for such an algebraic definition, tied to the imprecision of the previous work, as some students expressed: As such, we consider this comment as an ascendant proximity. To answer the teacher's question, some students then gave an intermediary expression: "The greater  $x$  is, the greater  $y$  is." The teacher insisted on the difficulty of this work, and it can be seen from the footage that some students of the class did not participate in it. The teacher tried to elicit something nearer to the targeted definition, but he was unable to get more from the students. It was apparent that he had to help them, and he suggested considering two values of  $x$  instead of considering all the values:

How can I see that  $x$  is increasing? I take greater and greater values of  $x$   
 ... I need to take at least two of them. How can it be written? How do we  
 discriminate between two different values?  $x_1$  and  $x_2$ . That is for  $x$ :  $x_1 < x_2$ .

Then, the teacher gave the complete expression and explained what it means graphically for the previously mentioned function (a descendant proximity). After this, the MTT continued, with decreasing functions and related topics.

#### *Differences between the MTTs in the two classes*

Finally, having analyzed each MTT, we can see the differences between them. However, it is important to note that we cannot explain the teachers' choices, as we do not know the full context of the classes. The greatest difference for us was the presence or absence of ascendant proximities, based on students' activities, to promote a link between their previous work on the variations of an explicit function and the general definitions of variations, including the difficult algebraic one. The second case (MM's class) showcased examples of these ascendant proximities, connecting at least the first ways to characterize an increasing function and the students' contextualized work. In the other class

(with GE as teacher), we noticed only descendant proximities, which we consider less likely to reach the students' ZPDs, and a possibly missed opportunity for an ascending proximity (which we had expected to take place). However, according to the difficulty of the algebraic definition and the gap between this definition and the students' previous knowledge, we must note that the algebraic definition had to be introduced by the second teacher himself, therefore with a descendant proximity only.

With these two cases, we have offered some evidence as to the analytical utility of our tools to study MTTs. The a priori study of the content at stake makes it possible for the researcher to identify opportunities for proximities. In this case, the study of highlights helped identify the need for a focus on the links between graphical and numerical knowledge and formal knowledge (algebraic formula) and the necessity to help students understand this difficult new formula, the need for it, and its meaning. Further, the study of the implementation of MTTs allows the researcher to detect what occurs in the classroom concerning these links and especially what remains implicit between contextualized and general knowledge. This suggests that, in the first classroom, some elements could have been developed further. However, we do not yet have evidence, other than theoretical, to ensure that the effects on student learning are different depending on the type of proximity. That is the next step of our research.

## CONCLUSION

We have developed in this article some new extensions of our previously constituted theoretical framework (Robert & Rogalski, 2005): Instead of analyzing students' activities directly, which are difficult to characterize during MTTs, we infer some possibilities as to students' understanding of the content at stake, drawn from specific comments in the teacher's discourse, called proximities. We particularly focused on highlighting the dynamics between general knowledge and its uses, which are usually at stake in MTTs. This involves the connections between the teacher's choices of (a) mathematical content; (b) content implementation during the sessions; and, maybe, (c) previous tasks and the anticipation of later work on other tasks.

One way of interpreting our work is to consider that these proximities, along with the exercises proposed after the MTT, may help students do the expected transformation of *pseudoconcepts* (i.e., what appears to be a concept without actually being one as it lacks foundational reasoning) into concepts (Vygotsky, 1978). It was this notion which inspired us to conduct this study. Of course, unlike Vygotsky (1978), our focus is with students and scientific concepts, not with young children and mundane concepts (i.e., concepts stemming only from experience), but we do suggest that what occurs during MTTs is similar. First, students hear words and see formulas, although they still might not attain the whole meaning, just as if they were considering a pseudoconcept. Then,

through various exercises where students put this information to use, and after their errors are revised and commented upon by the teacher, it is gradually transformed into a proper concept. It is as if, after the MTT, the students have in mind a partly empty envelope, with a label only, with the aim being to fill it with effective mathematics to be used and appropriated in a conceptual way. Adopting this metaphor, the importance for the teacher to provide and make use of all occasions to connect the objects (words, formulas) and their uses in context becomes evident – without neglecting the importance of the ZPD of each student in the learning process.

### *Some developments in our research using proximities*

#### *Widening the analysis*

First, it seems necessary to study more of these moments, in relation to other mathematical concepts, to be able to establish some regularity in the teaching practices concerning MTTs. Does a teacher always develop the same type of MTT for different mathematical content in one class? How about in different classes? Are there differences in a teacher's choices in regards to the students' age? Can we characterize differences between teachers according to the tasks they give to their students before and after the MTT, and to the content and organization of their MTTs? Some pieces of research have already shown differences between MTTs to introduce a concept (e.g., Horoks, 2006; Chesnais, 2011), but without the precise, localized study of proximities.

The main question for us is the potential effects of the choices for MTTs on students' learning during the lessons and perhaps after. Are there tasks that provide more or less opportunities for student reflection on which to build proximities? Are all students equally receptive to proximities?

#### *Individual and classroom ZPD*

Let us comment briefly on the open question of the relation between individual ZPD and what could be called a "classroom ZPD" (cf. whole-class scaffolding, Smit et al., 2013).

In our current research work generally, our observables for studying students' activities are mostly based on what is happening and what may be seen or heard in the classroom, and rarely on what occurs for individual students or subgroups. Teachers' discursive proximities can then be considered a means of making students move toward new conceptual knowledge, but the process refers to the class as an entity. Does the concept of ZPD remain relevant to analyze what could be considered as the teacher's scaffolding of a class-wide conceptualization? There can be several potential better knowers in the class context. During MTT, these students may scaffold other students' activities and (short-term) learning processes. In fact, a concept may be considered to be in the classroom ZPD, even if there are only some students for which the concept is



in *their* ZPD. This means that the mathematical activity, and the comments on the activity, of these students and their teacher might trigger the development of the other students' ZPD, and subsequently their knowledge. This question clearly necessitates more research.

Nevertheless, we do not know what potential effects we can expect on learning, nor can we easily distinguish MTTs' impact from the impact of the whole teaching process.

*Returning to teaching practices*

In the context of ordinary classrooms, there is a common question about teacher's comments: Is one telling too much, or perhaps telling too little? Making systematic discursive proximities during MTTs may appear to some students as boring or overwhelming, with too many comments. For others, it may help to reinforce their understanding (perhaps especially if they present a low level of self-confidence). Adapting to students' diversity when implementing the prepared proximities is a teaching challenge, requiring sensitivity to the reactions of the students in a phase (the MTT) where their activity is often hidden (this challenge concerns the "sensitivity to students" in Jaworski's triad; Potari & Jaworski, 2002, p. 352).

Finally, in addition to a new theoretical scope, the introduction of a new tool that operationalizes the concept of ZPD could have implications in terms of teaching practices and teacher education. Future research and study could bear out the utility of proximities for the classroom and, ultimately, for the students.

NOTES

1. For us, mathematical activities represent not only what students do or say, but also what they think.
2. We do not ignore their divergences regarding the philosophical background, the role of language in child development, and the relationships between learning and development.
3. However, we do not refer to the systemic view on activity theory as developed by Engeström and Sannino (2010).
4. The word "general" is often used by teachers themselves to describe the text of certain definitions, theorems, properties, formulas, methods, or even vocabulary exposed during MTTs. The word "decontextualized" may be used instead of "general" to highlight the fact that a piece of knowledge can be formalized independently from a particular context, outside of a specific problem; in such a case, it is more difficult for students to link it to what they already know or have done.
5. This issue is probably handled differently by different teachers.
6. It has also been introduced for teacher training as the *zone of proximal professional development* (ZPPD; Abboud et al., 2020).

7. We call a “proximity” (Robert & Vandebrouck, 2014; Bridoux et al., 2016) any element in the teacher’s decisions or discourse that (potentially) contributes to fill the gaps for (some of) the students, between the mathematics at stake and the students’ understanding. In our definition, they are both “cognitive proximities,” as they are supposed to have an effect on understanding, as well as “discursive proximities,” as they appear in the teacher’s discourse. There may be other kinds of proximities, such as a “proximity-in-act” when the teacher changes the task so that more students can work on it.
8. In fact, this label has not been introduced solely for the study of these moments, but it appears to be a privileged tool to perform this analysis.
9. Not strictly mathematical discourse but discourse *about* mathematics.
10. As supposed by the researcher.
11. We did not know what occurred during the work on the introductory task beforehand as it was not filmed.

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