ON THE ASSOCIATION BETWEEN MODES OF MENTAL REPRESENTATION AND MATHEMATICS EXPERIENCE IN TEACHER EDUCATION

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ABSTRACT. The purpose of this study is to examine one aspect of the cognitive development of preservice education students; i.e., the ability to utilize different modes of mental representation. This study attempts to provide a basis for understanding the relationship between the degree of experience in mathematics and the ability to utilize different modes of mental representation, using the "Modes of Thought Questionnaire" (MOTQ) of Aylwin (1985).

In the study, associations were found between the level of experience in mathematics and both the ability to utilize each mode of mental representation and the overall use of the preferred modes. This study has implications both for future research and for education. First, it gives a clear indication that differences do exist between the mental representation modes preferred by individuals with no mathematics experience, as compared to those who have even a small level of experience in mathematics. Second, it provides implications regarding teacher education in Ontario, which arise from these differences in the utilization of all of the modes of mental representation and knowledge acquisition.

RÉSUMÉ. Cette étude traite d'un versant du développement cognitif des professeurs stagiaires, c'est-à-dire de la capacité à utiliser différents modes de représentation mentale. Elle vise à établir les fondements permettant de comprendre la relation qui existe entre le niveau d'expérience des mathématiques et la capacité d'utiliser différents modes de représentation mentale mesurée à l'aide du questionnaire MOTQ d'Aylwin (1985).

On a constaté qu'il existe un lien entre le niveau d'expérience des mathématiques, d'une part, et la capacité d'utiliser différents modes de représentation mentale et l'utilisation globale des modes de prédilection, d'autre part. Cette étude aura des répercussions sur les recherches et l'éducation. Elle révèle tout d'abord qu'il existe des différences entre les modes de représentation mentale que préfèrent ceux qui n'ont aucune expérience des mathématiques et les modes que privilégient ceux qui possèdent un peu d'expérience dans ce domaine. L'incidence que cela pourrait avoir sur la formation des enseignants en Ontario tient aux différences observées dans l'utilisation de tous les modes de représentation mentale et l'acquisition des connaissances.

There is a cycle of lack of success in mathematics study in many school systems. That is, a student who is having difficulty in learning mathematics may cease to study this discipline, but a few years later become a teacher, and be obliged to teach mathematics. The students attempting to learn mathematics in such a teacher's classroom may then have difficulty in learning mathematical topics. Hence the cycle continues.

An opportunity to break the cycle exists during the preservice education year at faculties of education. In order to do this, however, fundamental research must first be carried out into the nature of the lack of success of mathematics. The long-term persistence of this problem is reflected in the fact that, despite all the changes in teaching methods and curricula over the years, and despite the current stringent entrance conditions to the education program, little improvement has been noted. Hence researchers are attempting to change the focus of research, to examine more personal factors such as that of cognition, where this research is situated. In particular, this study attempts to compare the degree of mathematics experience of the preservice education students to their utilization of different modes of mental representation.

NATURE OF THE STUDY

Each preservice education student comes to teacher education with a history of mathematics experiences which may have been positive or negative in nature. A student may have been successful or not successful in the mastery of various mathematical topics or concepts as presented in academic courses. Students registered in the one-year program are aware that certification in Ontario, as elsewhere, presupposes competence in mathematics to the grade 10 level, and hence know that the teaching of mathematics will be an integral part of their teaching responsibilities. Even further, some of these students may be required to teach mathematics content which they themselves avoided in school, such as the solution of equations, geometry, or percentage. As Park (1990) stated, some *preservice* education students may not have had any mathematics experience since grade ten. In his words, "They teach mathematics and science because they are required to, not because they have special training" (1990, p. D2). Each student has also developed an individual approach to learning, one aspect of which is the utilization of modes of mental representation. It is the association between experience in mathematics and utilization of modes of mental representation which the present research attempts to explore.

The difficulty with modes of mental representation is that one cannot be sure that the model being used is correct. The cognitive models which theorists have developed are based on the external actions of individuals, and the nature of the mental processes behind the actions cannot be easily discerned. These models attempt to explain how the continuous barrages of sensory stimuli that assault the individual are configured into understandable forms which become personal models of reality (Agnew & Brown,1989; Howson & Kahane, 1990). This selective attention to stimuli, and the resulting creation of mental models of reality, form the basis of constructivist theories.

Many theorists in this field appear to present views which are consonant with Jerome Bruner's theory of cognitive development. One of these, Susan Aylwin, has written a clear, concise description of modes of representation and how they develop. Her work is important for two main reasons: first, her concept of mental representation may have a bearing on the differing abilities of students to master the concepts required in the study of mathematics; and second, she has developed empirical instruments which can be used to evaluate the different forms of mental representation used by individuals.

Aylwin, along with others (Bruner, 1964; Horowitz, 1972), posits three modes of mental representation that are sequentially acquired in the developmental process of individuals. Aylwin's modes are reminiscent of those of Bruner's enactive, iconic, and symbolic forms of mental representation, except that she considers them to be mental images (1985). She calls the first form of representation "enactive imagery", and states that this form is kinaesthetic in nature, involving muscular tension in the same way as Bruner's enactive form of mental representation. It involves, according to Aylwin, imagined action and role playing. Her second form of representation is that of "visual imagery". She uses the "pictures in the mind's eye" metaphor to describe this representation, and suggests that there are parallels between the mental processing of a stimulus being viewed by the eye, and of mental images, either of things seen previously and remembered, or of completely imaginary things. The third form of mental representation is, for Aylwin, "verbal imagery". This is a form of inner speech, with the same properties as outer speech. It is used to code information, and to handle conceptual and abstract thought. She states Vygotsky's view, that there are structural differences between inner and outer speech, in that inner speech is highly contracted, and contains mostly predicates without subjects. Verbal imagery, by this definition, consists of words and sound, and is structured categorically and hierarchically.

The actual proficiency level demonstrated in the use of each representation differs amongst individuals. Some use more enactive levels of thought, some use more iconic, and some use more symbolic. The majority of people use all three forms in different circumstances, but to differing degrees. Aylwin makes the further conjecture that the three modes of mental representation may use different cognitive structures, and hence may have different kinds of affects and feelings associated with them (1985). She suggests, moreover, that this gives rise to three personality styles, namely verbalizers, visualizers, and enactive imagers.

The modes of thought questionnaire

Aylwin (1985) states that adults, in general, have personal preferences in the utilization of modes of representation to solve specific tasks. She has developed an instrument to assist in her research into mental representation, called the MOTQ *or Modes of Thought Questionnaire*. The purpose of the instrument is to evaluate individual preferences in modes of representation, and hence to assess representational biases through their "characteristic cognitive structures" (p. 66). In other words, Aylwin measures, with the MOTQ, the ability of an individual to comply with directions to utilize particular modes of mental representation, and she observes the actual modes used in the process.

Aylwin has used the MOTQ to examine the biases evinced towards the modes of representation by students preparing for different careers. A strong bias for verbalization was found in commerce students; visualizing was found to be associated with arts and social science students, particularly women; and engineering students were found to have a bias for enactive imagery.

The relationship between preferred modes of representation and careers is interesting in light of the current question, concerning the possibility that the relative ability to utilize the three modes of mental representation has a bearing on the ability of a student to master mathematics in general, and specific topics within the discipline, such as geometric proof, or the solution of three equations in three unknowns. To examine this relationship in greater depth, it is necessary to examine the three modes of mental representation more closely.

Aylwin considers each mode of representation to be a way of thinking, in that a particular representation of reality directs one's thinking into a path related to the representation. Each mode of thought can be understood as being a way of making sense of the world, in that paying attention to the same object, but using different modes of representation, may result in different outcomes. These different ways of making sense of the world by individuals who use different modes of representation have an important implication for the present research. For example, in school-based learning, it may be that the mode of representation used by the student influences the understanding of and response to different kinds of lessons presented by teachers, as well as the degree of attention paid to specific subject material. Moreover, the degree of learning taking place in classrooms may be influenced by the degree of similarity between the modes of mental representation used by students and teacher.

One can consider, for example, the mathematical problem of finding the distance across a lake without employing direct measurement. The visualizer might focus on the lake itself, and look in depth at a mental image of the lake in question, to find clues to the problem solution. The verbalizer, on the other hand, might imagine a hierarchy of problem-solving techniques, in which "lake, distance across" would be a category, and various methods of solution would be written in a table. The enactive imager might imagine the physical process of taking measurements, walking, and so on, to determine a method of solution. Hence a different preference in mode of mental representation between individuals may reflect a different approach to problems. The teacher who thinks only of one type of solution will not necessarily "get through" to students who have other frames of reference.

More specifically, it is plausible to suggest that achievement in school subjects preferred by students may depend upon their ability to represent the content in the appropriate mental modality. Overuse of a preferred mode of mental representation, according to Aylwin (1985), Rancourt (1978), and Royce (1964), may bring a student to handle easily only one of the possible three modes. Hence students who do not succeed in mathematics may show a particular hierarchy of use of the three modes of mental representation.

Statement of hypothesis

The hypothesis that arises from the theoretical framework and the review of the research literature on the construction of mental representation can be stated as follows: There is an association between levels of experience in mathematics and the degree of effective utilization for each of the three modes of mental representation, as measured by Aylwin's MOTQ (1985).

METHODOLOGY

This study examines the mental representation of preservice education students in Ontario. The instrumentation, Aylwin's MOTQ, is designed to determine individual preferences in the utilization of the three modes of mental representation. The questionnaire is concerned with how a subject responds to the associations between ideas. It examines the underlying assumption that thinking in different modes results in different associations.

The MOTQ is a word association test, in which the subject is directed to think in a particular way for words that provide a specific stimulus. For example, to encourage an individual to use the visual form of mental representation, instructions are given to the subject to see a picture in the mind's eye after being presented with a stimulus word, and then to write down the first association to the image which comes to mind. Each response is then coded as to mode of mental representation utilized. For example, if the stimulus word is "baboon", then a response of "a great ape" would be classified as verbal; a response of "shaggy fur" would be classified as visual, and a response of "afraid" or "I'm swinging in the tree" would be classified as an enactive response. Some associations, however, particularly synonyms, cannot be coded, since it is not possible to distinguish the mode of mental representation employed in making the association. The stimulus words are carefully chosen so that they may evoke different responses, for the test is an attempt to find out whether or not the subjects can continue to use a directed mode of mental representation over a period of time.

The data resulting from the MOTQ consist of numerical scores of the utilization of each of the three modes of mental representation, for each section of the questionnaire. There are data concerning the utilization of a mode when told to do so, and also the utilization of one mode when requested to use a different mode. There are 24 stimulus words in the MOTQ for each of the three modes of mental representation, and a subject who is very adept at all three ways of mental representation can theoretically achieve a score of 24 on each of the three modes. In practice, however, this does not often occur. Rather, one mode of mental representation tends to dominate to some extent, and scores on any one mode may vary widely.

The MOTQ has been used previously to determine the preferred hierarchies of modes of mental representation of Australian university students in the fields of engineering, commerce, and the arts. In all, more than 2,000 students participated in studies using the MOTQ. The results have been found to be reliable, in that the findings correlate well with research from Witkin, Ashton, J. Richardson, and others (Aylwin, 1985; Richardson, 1969; Witkin & Goodenough 1981). The test-retest reliability ranges from .69 to .84 on the three aspects of the instrument, for a seven-week period, while the Cronbach alpha measures range from .83 to .89 on the three aspects of the instrument. Construct validity was tested using principal axis factor analysis on the subscales. The highest loading subscales on each factor were, indeed, those appropriate to the relevant mode of representation. In an earlier survey of instruments testing modes of mental imagery, White, Sheehan, and Ashton (1977) found a surprising agree of reliability for word association tests of modes of mental imagery, while some researchers (Richardson, 1969) make the connection between mental images and mental representation.

Two different versions of the MOTQ are actually used in this study: the MOTQ-1 and the MOTQ-2. They differ in two ways: first, that the MOTQ-1, the original instrument, requires triple the amount of time to complete, and second, that in the MOTQ-1, the subjects are asked to judge the probability level which they would give to pre-determined word associations; whereas the MOTQ-2 allows subjects to give any word association. This second difference is important, in that many phrases in the MOTQ-1 are specific to British or Australian culture, in that terms such as "badgers" or "traffic wardens" are not common words in the vocabulary of North Americans. The MOTQ-2, on the other hand, allows a subject to make any desired association, and hence is more acceptable.

Since the psychometric data concerning the MOTQ instrument were evaluated using the MOTQ-1 (Aylwin, 1985), a small subsample of approximately 35 subjects completed this questionnaire in addition to the MOTQ-2. A delay of six months was allowed between the completion of the two instruments, since it was felt that there would be a risk of remembered responses. The results provided by the two MOTQ questionnaires were compared. The Pearson product-moment correlations between the two MOTQ questionnaires were .60 for the Verbal mode, .62 for the Visual mode, and .53 for the Enactive mode. The correlations between the two MOTQ questionnaires appear not to be as high as one might expect for two tests which purport to measure the same modes of mental representation. The range of correlations found, however, must be considered in the light of the following two factors. First, some of the vocabulary of the MOTQ-1 was indeed confusing to the subjects, since they were not of British origin. Second, allied to the first, is the fact that the majority of subjects rated some common British associations as being very unlikely, although these were listed as being very likely on the coding charts. In examining these data, the phrases which were unknown to the students are precisely those which bring the correlation down.

The MOTQ-2 was selected, therefore, because of the freedom from such cultural bias, in that any association made by the subjects could be coded regardless of language used, providing that a possible coding existed. This was necessary, since the students in Ontario faculties of education reflect the multicultural reality of the province today.

Sample description

The subjects who participated in this study were taken from the population of preservice education students registered in various 1990-91 Bachelor of Education programs in the province of Ontario. These are one-year programs following an undergraduate degree. Permission to conduct the study was received from the universities concerned, in the Eastern and Mid-North regions of Ontario. Participation in the study was on a volunteer basis, and was invited according to the general guidelines set by a university ethics committee.

A total of 223 subjects took part in the study. The responses of eight subjects were discarded because they either did not understand the instructions, did not complete both questionnaires, or did not identify themselves. The final sample consisted of 215 subjects. Of that total, eighty-six were planning to teach younger students in grade six or lower, while 129 were planning to teach older students in grade seven or higher. The gender breakdown of these subsamples is provided in Table 1, as is the breakdown by mathematics experience, age, subject major at university, and experience in mathematics. The numbers in this table reflect the fact that four of the 215 subjects did not indicate gender during the testing session.

The ages of subjects ranged from 20 to 54, with a mean age of 28.3 years and a median age of 26. Approximately one third of the subjects came directly into *preservice* education studies after graduation from university training (37.1% were under 25 years of age), while approximately one in every six subjects (16.7%) were more than 34 years of age, and hence had a considerable gap between university training and entrance into the faculty of education.

Various samples of subjects were formed from the data. The main set of subsamples was formed on the basis of different levels of experience in mathematics, in order to test the hypothesis. A second set of subsamples was also formed based on programs of study: both those followed during post-secondary education of subjects, and those followed during the year of teacher education. The subsamples formed according to programs of study during post-secondary education were used in a more specific analysis than that possible with the main sample. This analysis provides additional support for the findings from the main sample with respect to the hypothesis.

The criteria for the placement of subjects in the sample of "degree of mathematics experience" were as follows. The first subsample, the "no experience level in mathematics" subsample, consists of subjects who have no mathematics experience of any type at the post-secondary level. They have not studied mathematics, nor have they studied any related disciplines such as science, nursing, engineering, or kinanthropology which use mathematics in their programs of study. The fourth subsample, the "high mathematics experience" subsample, consists of subjects who have taken at least 6 courses in mathematics at the post-secondary level. These two subsamples comprise approximately half of the total. The remaining subjects in the sample are differentiated by means of their mathematics experience, into two subsamples respectively: by the study of mathematics directly, and by the study of courses which contain a considerable mathematical content. Such courses are taken into consideration in the subsample composition, so as to provide a means by which the two middle subsamples, the "low experience level" and "moderate experience level" subsamples of mathematics experience, can be determined. The breakdown of the sample with regard to mathematics experience is shown in Table 1.

In this table, the four subsamples are seen to be approximately equal in size, with percentages of subjects ranging from 23.7% to 27.9%. The subsamples formed with respect to mathematics experience are also found to be similar to one another with respect to age and experience in the workplace. The second method of grouping by mathematics experience involves the subsamples formed according to university majors. These provide a different way of examining the effect of math-

TABLE 1. Description of subjects (n-215)

BREAKDOWN BY GENDER AND PROGRAM OF STUDY

Program of study	Male		Female		Total	
	N	%	N	%	N	%
Grades 1 to 6	15	7.1	70	33.2	85	40.3
Grades 7 up	56	26.5	70	33.2	126	59.7
Total[a]	71	33.6	140	66.4	211	100.0

BREAKDOWN BY MATHEMATICS EXPERIENCE AND AGE

Mathematics Experience	N	%	Age Range	Average Age	Median Age
None	60	27.9	22 - 54	29.2	27
Low	51	23.7	22 - 44	28.8	26
Moderate	53	24.7	21 - 50	27.2	25
High	51	23.7	20 - 54	27.8	24
Total	215	100.0			

BREAKDOWN BY UNIVERSITY MAJOR AND MATHEMATICS EXPERIENCE

University Subject Major	N	%	Average Number of Courses Taken Containing Mathematics: Math Only /	Math+Related
Languages	26	12.5	0.10	0.9
Humanities	50	24.0	0.72	6.1
Physical Ed.	23	11.1	2.80	13.9
Life Science	39	18.7	4.17	18.4
Science	28	13.5	4.74	20.8
Mathematics	42	20.2	9.40	26.7
Total[b]	208	Average	3.80	14.8

[a] - 4 subjects did not indicate gender

[b] - 7 subjects did not have any of these majors of study

ematics experience, since different university majors involve different numbers of courses in mathematics and mathematics-related courses, as illustrated in Table 1. Of the group who have studied languages, for example, a total of one subject in ten has studied one course in the mathematics department, leading to an average of 0.1 course for the group. Nine out of ten of these individuals, then, have taken no courses at the post-secondary level such as science, nursing, and kinanthropology, which contain any mathematics at all.

Whether one lists only the actual mathematics courses taken, or whether one includes the courses such as physics or geological science which involve the use of some mathematics, the programs of study sort into the same order, with the language majors having the least experience in mathematics, and the mathematics majors, the most experience in mathematics. In the third portion of Table 1, the total number of subjects is 208, since the remaining seven subjects, mainly music majors, are too few in number to form a viable group.

Statistical methods used in data analysis

Various statistical techniques are utilized in the analysis of data in this study. In the examination of categorized data such as that derived from frequencies of subjects with particular dominant modes of mental representation, the nonparametric procedure of Chi-Square is used. Contingency tables are formed to examine various subsamples on the basis of the dominant modes of mental representation. Multivariate analyses of various kinds, including multiple regression and principal component analysis, are also carried out on these data.

In a further analysis of individual scores achieved in the subscales of the different instruments, the scores are treated as continuous data, by use of the parametric ANOVA procedure. The ANOVA procedure is used primarily to test for significant differences in utilization of the three modes of mental representation as measured by the MOTQ in the groupings by university major, and by mathematics experience. Wherever the null hypothesis is rejected, post hoc analysis is carried out by means of Scheffé's procedure, a robust method of analysis for unequal sample sizes such as those present here.

RESULTS AND DISCUSSION

With regards to the MOTQ, there are two aspects to consider in the examination of the "degree of effective utilization" of the modes of mental representation. In the MOTQ, the subjects are directed to utilize each of the modes in specific portions of the test. If a subject is able to utilize a particular mode when directed to do so, then the subject should utilize the visual mode on the visual portion of the test, and so on. For this reason, the first aspect of effective utilization is the degree of compliance to an instruction to use a particular mode; that is, the degree of individual control. The second aspect of effective utilization to consider is the personal preference of use of each mode of mental representation by subjects, regardless of instructions given.

TABLE 2. Utilization of modes of mental representation

	MATHEMATICS EXPERIENCE GROUPS							
Degree of compliance	1		2		3		4	
to direction to use	NONE		LOW		MODERATE		HIGH	
each mode	N	%	N	%	N	%	N	%
Uses directed mode	23	38.3	27	53.0	33	62.2	40	78.4
Does not use directed mode	37	61.7	24	47.0	20	37.8	11	21.6
Totals	60	100.0	51	100.0	53	100.0	51	100.0

The actual ability on the part of a subject to utilize a particular mode of mental representation, when directed to do so, is not consistently demonstrated in the data. In Table 2, the percentage of subjects with the ability to do this is recorded. If the subject uses each of the three modes as directed, then the correct dominance is said to occur; otherwise, the subject is said to have incorrect dominance. The threshold of correct dominance for each mode is set at 80% use of the directed mode, since the percentage of subjects using the appropriate mode remains approximately the same (between 80% and 95%), while few subjects have greater accuracy.

In Table 2, the degree of direction compliance to utilize a particular mode of mental representation is presented in relation to the "levels of experience" in mathematics of each subject. The results of testing for appropriate use of the three modes of mental representation are shown to be mixed, in that many subjects do not use the particular mode which they are instructed to use. In this table, one can note that the percentage of subjects utilizing all three modes, as directed to do so, increases as the amount of experience in mathematics increases. The percentages of subjects utilizing these modes as directed, range in a steady progression from 38.3% for the subsample with no mathematics experience, to 78.4% for the subsample with considerable mathematics experience.

The findings displayed in Table 2 are examined by means of a four by two contingency table, comparing mathematics experience to correct and incorrect mode utilization. At the 0.01 level of significance, an association is found to exist between levels of mathematics experience and the appropriate utilization of the modes of mental representation (Chi-square = 15.73, df = 3). The chief difference is found in the two extreme subsamples: the "no experience level" of mathematics, and the "high experience level" of mathematics. In these, significantly more subjects in the subsample with a high level of experience in mathematics are able to utilize the modes as directed, than in the group with no math experience.

The second aspect of "effective utilization" of the modes of mental representation, namely that of overall utilizations of each mode of mental representation, is examined in Table 3 for the same four subsamples. For example, the number of times that the verbal mode is used is recorded without regard to the requested mode. The average raw score of the majority of subjects is highest in the visual mode, as expected from the history of the MOTQ, since in Aylwin's (1985) work, students in the social sciences, and women in general, have been found more likely to have visual dominance. In this study, the sample contains many subjects who fall into these two categories. The degree of utilization of the three modes of mental representation for each subject is therefore calculated by comparison with the average score on each particular scale, in order to accentuate the differences among the subsamples of subjects. The dominant mode is defined as the mode in which the score of the subject is farthest away from the mean score: in other words, it is the mode used most frequently, as compared to the mode used by the average subject. It should be noted, moreover, that this method of calculation for the dominant mode provides the same statistical results as does the use of raw scores, but the information is presented in a more accessible form.

TABLE 3. Mental representation and mathematics experience

	DOMINANT MODE OF MENTAL REPRESENTATION							
Mathematics Experience	VERBAL		VISUAL		ENACTIVE		TOTAL	
Level	N	%	N	%	N	%	N	%
None	5	11.1	28	62.2	12	26..7	45	100
Low	17	42.5	6	15.0	17	42.5	40	100
Moderate	20	44.4	7	15.6	18	40.0	45	100
High	12	33.3	8	22.2	16	44.4	36	100
Total	54	32.5	49	29.5	63	38.0	166	100

As shown in Table 3, verbal dominance is observed to range from 11.1% in the "no experience level" subsample, to 44.4% in the "moderate experience level" subsample. Visual dominance ranges from 15% in the "low experience level" subsample to 62.2% in the "no experience level" subsample. Finally, enactive dominance ranges from 26.7% in the "no experience level" subsample, to 44.4% in the subsample with the "high experience level" in mathematics.

The subsample with the "no experience level" in mathematics appears to stand apart from all other subsamples. This difference does not appear to be related to the level of mathematics experience, or one would expect a steady progression in scores. This does not occur, but rather it appears that the three groups with math experience are alike, no matter what the level of experience.

The results from Table 3 are examined by means of a four by three contingency table. At the 0.01 level of significance, an association is found to exist between "level of experience" in mathematics and dominant mode of mental representation (Chi-square = 34.5, $df = 6$). More specifically, the subsample of subjects with the "no experience level" in mathematics is found to differ from the other three subsamples in terms of use of the verbal and visual modes of mental representation. No significant difference is found, however, among the subsamples regarding the enactive mode of mental representation.

Each of the three mode scores is examined separately using multiple linear regression analysis with variables age, gender, level, and percentage of courses in mathematics, science, life science, and the humanities. The result confirms the findings of the ANOVA analyses, and provides a little information concerning one or two other variables. In particular, age is found to be of slight statistical importance to the visual mode score ($p = 0.022$), and science is found to have a slightly greater impact on the scores than does mathematics. Principal component analysis also confirms the findings from the other statistical analyses, but does not add new information.

DISCUSSION OF FINDINGS

The research hypothesis, that there is an association between levels of experience in mathematics and the utilization of different modes of mental representation, cannot be rejected. Specifically, a lack of experience in mathematics appears to be related to an inability to utilize correctly all three modes of mental representation when directed to do so (Table 2). Furthermore, a high level of experience in mathematics is related to a relatively high degree of ability to utilize all three modes of mental representation when directed to do so. The differences among the three groups with some mathematics experience are not, however, statistically significant, although all three groups are significantly different from the group with no mathematics experience on this measure. The subsample of subjects with no mathematics experience is found, moreover, to have a statistically significantly lower utilization of the verbal mode of mental representation, and a higher use of the visual mode, than do the other subsamples. In summary, then, the research hypothesis is confirmed: that there is a difference in degree of effective utilization of the three modes of mental representation which is associated with level of experience in mathematics. The greatest difference is found in the use of the verbal mode, a mode associated with organization and logic.

There is, however, no significant difference in the ability to use a particular mode of mental representation among the three subsamples with some mathematics experience. Even the subjects in the subsample with low experience in mathematics, however, have up to 15 post-secondary courses in subject majors which require some expertise in mathematics, such as physics, computer science, and kinanthropology. The subjects in the subsample of "moderate experience level" in mathematics have, moreover, taken from 16 to 34 courses in subjects which require a considerable degree of mathematics experience.

It is therefore possible that the mathematics studied in a mathematics course, and mathematics studied within a physics, or other non-mathematics course, both provide similar practice in the uses of the different modes of mental representation. For example, the understanding required to calculate the trajectory of a javelin in a kinanthropology course may not differ from the understanding required to calculate velocity of a projectile in a calculus course. Only the frame of reference is changed; the problem is the same.

It can be argued, then, that mathematics experience that is obtained solely in mathematics courses does not differ from mathematics experience in related courses. This, however, was not examined by Aylwin, who was interested in broader differentiation amongst areas of study.

Exploratory analysis

The results described above, concerning the hypothesis and the MOTQ, are those resulting directly from the research. There are, however, other findings of interest to this hypothesis, which arise from the data generated during the study. The MOTQ is not designed solely to identify the dominant mode of an individual. Rather, as Aylwin states, it "assumes that each person uses all the forms of representation, with the balance varying between individuals" (1985, p. 67). Hence, in the examination of the utilization of the modes of mental representation, the scores achieved on each of the subscales of the MOTQ were next examined, rather than just the dominances.

The data on scores achieved on the subscales of the MOTQ permit a deeper analysis of the major differences between subjects who have prior experience in mathematics, and those who do not. Table 4 provides the mean scores and their standard deviations, on each of the three modes, for each of the subsamples formed with respect to mathematics experience. The average scores of the subsample with no experience in mathematics are lower on the verbal and enactive scales, and higher on the visual scale, than are the scores of the other three subsamples. Table 4 indicates again that there does not appear to be a progression in scores as experience in mathematics increases; in fact, the scores achieved on the three scales, by the three subsamples with at least some mathematics experience, are very similar, as shown earlier in Table 3.

TABLE 4 Means and standard deviations of the mathematics sub-samples on the MOTQ

		MODE OF MENTAL REPRESENTATION					
Mathematics		VERBAL		VISUAL		ENACTIVE	
experience	N	M	SD	M	SD	M	SD
None	60	17.87	5.45	22.67	4.30	18.07	6.68
Low	51	21.30	5.05	18.71	3.30	20.41	7.28
Moderate	53	21.28	5.73	19.00	4.15	20.61	5.17
High	51	20.07	4.22	19.30	3.32	21.24	4.80

The three modes shown in Table 4 are examined separately for the four subsamples of subjects organized with respect to mathematics experience. As earlier in Table 3, it appears that the subsample of subjects who have not studied any mathematics, either in mathematics courses, or indirectly, as part of other courses, portray a different profile than do the subjects in the subsamples "low experience level", "moderate experience level", and "high experience level" in mathematics. The scores achieved in the "none level of experience" mathematics subsample appear lower on the verbal mode and enactive mode, and higher on the visual mode, than do the scores of the other three subsamples.

The scores achieved on each of the three subscales, for each of the four subsamples, are analyzed using a series of one-way ANOVAs, shown in Table 5. The one-way ANOVA of the verbal mode scores demonstrates that the means achieved by subjects among these four groups are distinct at the 0.05 level of significance, and thus that, as a whole, the subsamples are not all taken from the same population.

TABLE 5. ANOVAs of scores on the MOTQ by mathematics subsamples

ANOVA OF MOTO VERBAL SCORES

SOURCE	DF	SS	MS	F	P
Math Group	3	414.8	138.3	4.88	0.003
ERROR	211	5973.4	28.3		
TOTAL	214	6388.2			

```
Mathematics        95% CONFIDENCE INTERVALS FOR GROUP MEANS
Experience    --------+---------+---------+---------+-
   None        (-----*------)
   Low                          (------*-------)
   Moderate                    (------*------)
   High                   (------*------)
              --------+---------+---------+----------+-
                    18.0      20.0      22.0
```

ANOVA OF MOTQ VISUAL SCORES

SOURCE	DF	SS	MS	F	P
Math Group	3	592.2	197.4	12.78	0.000
ERROR	211	3258.2	15.4		
TOTAL	214	3850.4			

```
Mathematics        95% CONFIDENCE INTERVALS FOR GROUP MEANS
Experience    --------+---------+---------+---------+---
   None                                  (----*----)
   Low             (-----*----)
   Moderate          (----*----)
   High               (-----*----)
              --------+---------+---------+---------+---
                    18.0      20.0      22.0      24.0
```

Post hoc analysis, using Scheffé's procedure, reveals that the subsample with no mathematics experience has significantly lower scores on the verbal mode of mental representation than do the subsamples with low or moderate levels of experience in mathematics. The difference between the subsamples of subjects with no experience and with high experience, however, is not statistically significant. This is shown in the set of confidence intervals in Table 5, which provides a visual illustration of the differences among the subsamples.

This is interesting, in that most of the mathematics experience obtained by subjects in these two subsamples has occurred outside of the mathematics department. That is, the mathematics studied by these subjects has been to a large extent, applied mathematics, such as that in kinanthropology, science, or similar areas. The finding illustrated in Table 5 supports the suggestion made in the discussion of the hypothesis, namely that the effect of the mathematics studied in a mathematics faculty or department does not appear to differ from that of the mathematics learned in a course in a different department or faculty.

The visual mode scores are examined in the same way as the verbal mode scores. The mean of the subsample with no experience in mathematics is statistically different from the means attained by each of the other subsamples. The average scores of the subjects in this subsample are higher than those of the other subsamples. These data are illustrated in the confidence intervals of the subsample means, found in Table 5.

When the enactive mode scores are examined, it is found that the mean score of the subsample of subjects with no mathematics experience is lower than the scores obtained by the individuals in the other three subsamples, and that there is a steady progression of means scores from 18.07 to 21.24, as shown in Table 4, but that these differences are not found to be significant statistically.

The greatest differences lie, therefore, in utilization of both the visual and verbal modes by the subsample of subjects with no experience in mathematics as compared to the subsamples with low and moderate experience in mathematics. The subsample with the highest level of mathematics experience is not as far away as the two middle groups. These findings support the position that mathematics in itself does not provide the only answer for differing uses of the modes of mental representation.

There are two possible explanations for this: first, that perhaps mathematics is not the only factor playing a role in the use of the three modes of mental representation; and second, that the composition of the subsamples is perhaps too heavily weighted in favour of mathematics taught in mathematics courses, rather than in applied mathematics. It is even conceivable that the students of mathematics alone, with perhaps 10 courses, had effectively less exposure to the practical use of mathematics than science students with as many as 18 courses with a strong mathematical content.

Subsamples of post-secondary areas of concentration

The four subsamples formed with respect to mathematics experience are, in one respect, not as satisfactory as one would expect, since the separation between the subsamples is not clear-cut. In light of this, a re-examination of subjects as to academic profile was undertaken, in which an analysis of the program of study of each subject at the post-secondary level was done. The results indicate that the academic concentrations clearly differentiate the mathematics experience of the subjects, and hence provide a different focus for an examination of the relationship between mathematics experience and of utilization of the three modes of mental representation.

Six subsamples of subjects were formed according to the academic areas of concentration, or majors, at university. These areas of concentration are humanities, languages, mathematics, physical education, science, and life science. The subjects with other majors, such as music, are too few in number and hence are not included in this portion of the study. The results of the re-examination are provided in Table 6, wherein the results for the subsamples are listed in increasing order of average number of courses in mathematics studied at the post-secondary level. The numbers of such courses range from an average of 0.10 of a mathematics course for the subsample of subjects with language majors, to an average of 9.40 courses for the subsample of mathematics majors.

The confidence intervals of Table 6 illustrate a steady progression of means for most areas of concentration. The progression for the verbal mode means is an increasing progression, while the progression for the visual means is a decreasing one. *Post hoc* analysis on these findings, using Scheffé's procedure at a confidence level of 0.05, shows a significant difference between the scores of the language subsample and of the science subsample on both the verbal and the visual measures, and between the scores of the humanities subsample and the science subsample on the visual mode.

The subjects who studied languages at university are found to differ significantly from those who studied science, on both the Aylwin verbal and visual measures. On the verbal measure, the mean of the language subsample is 16.94, while that of the science subsample is 21.88. On the visual measure, the mean of the language subsample is 22.41, while that of the science subsample is 17.57. The subjects who studied humanities, on the other hand, differ significantly from the science majors on the visual mode, but not on the verbal. The enactive mode mean scores are also examined with respect to these subsamples of subjects. In this analysis, there is neither a progression according to mathematics experience, nor any significant differences among the subsamples formed with respect to post-secondary programs of study.

TABLE 6. MOTQ subscale scores vs post-secondary area of concentration

ANOVA OF VERBAL SCORES BY AREA OF CONCENTRATION

SOURCE	DF	SS	MS	F	P
subsample	5	408.6	81.7	2.81	0.018
ERROR	205	5964.4	29.1		
TOTAL	210	6373.0			

Area of concentration	N	VERBAL M	SD	95% CONFIDENCE INTERVALS FOR MEANS
Languages	25	16.94	5.42	(--------*-------)
Humanities	46	18.99	15.40	(-----*-----)
Phys. Ed.	25	20.09	6.42	(-------*-------)
Life Sci	43	20.76	5.47	(-----*------)
Science	28	21.88	6.47	(-------*-------)
Maths	43	20.83	3.53	(-----*------)
				15.0 · 17.5 · 20.0 · 22.5

ANOVA OF VISUAL SCORES BY AREA OF CONCENTRATION

SOURCE	DF	SS	MS	F	P
subsample	5	377.4	75.5	4.53	0.001
ERROR	205	3418.7	16.7		
TOTAL	210	3796.1			

Area of concentration	N	VERBAL M	SD	95% CONFIDENCE INTERVALS FOR MEANS
Languages	25	22.41	4.17	(------*-----)
Humanities	46	21.84	4.58	(---*----)
Phys. Ed.	25	20.30	4.30	(-----*-----)
Life Sci	43	18.97	3.84	(----*----)
Science	28	17.57	3.50	(-----*-----)
Maths	43	19.33	3.43	(---*----)
				17.5 · 20.0 · 22.5 · 25.0

The relationship between experience in mathematics and utilization of modes of mental representation is more evident with respect to the areas of concentration (Table 6) than is evident among the four subsamples formed with respect to levels of experience in mathematics.

Table 6 indicates that as mathematics experience in mathematics courses increases, so too does the use of the verbal mode; it also suggests that as mathematics experience increases, the use of the visual mode decreases. The progression in each case, however, is not completely smooth, in that the subsample of science majors is out of step. These subjects, despite having studied slightly fewer mathematics courses than those in the subsample of mathematics majors, have even higher verbal scores and lower visual scores than do the mathematics majors.

The main difference between those science majors and mathematics majors suggests that science majors include mathematics and science experience, whereas mathematics majors do not, in general, have much science experience. Hence, although mathematics experience appears to be related to the ability to utilize different forms of mental representation, it is possible that such experience is not the only factor involved. That is, the application of mathematics to other subjects may also be of critical importance in the development of the ability to use the verbal mode of mental representation.

The study of science and the study of mathematics can be considered to be an intersection of two sets of topics. That is, science courses and mathematics courses sometimes contain identical content. For example, the study of vector forces is the same in both algebra and physics courses, and modern set theory is now an integral part of modern chemistry as well as of algebra. Each of the two disciplines, however, contains some distinct topics unique to the discipline. The fact that the subjects in the subsample of science majors were better able to utilize the verbal mode, and less able to utilize the visual mode of mental representation, than were the subjects in the subsample of mathematics majors, suggests that these discipline-specific topics are of importance.

Further research should be undertaken to find out more about these factors which are involved in science, yet not involved in mathematics. Research should also be undertaken to examine specific experiences in the study of different branches of mathematics; for example, pure mathematics, applied mathematics, and statistics, as well as in the application of mathematics to other areas of study.

IMPLICATIONS FOR THE FUTURE

This study has clear implications both for future research and for education. First, it gives a straightforward indication that differences do exist between the mental representation modes preferred by students enrolled in teacher education who have no mathematics experience, as compared to those who have even a small amount of experience in mathematics. Second, it implies that future research should concentrate on the differences in teaching which may arise from these differences in the utilization of all of the modes of mental representation.

That is, since we now know that student teachers who have no mathematics experience differ from the rest of the student population both in their use of the modes of mental representation and in their preferred knowledge accessing modes, the next question for research is that of the effect, if any, of this difference on the required teaching of mathematics which these individuals must conduct in the performance of their duties. As Ebenezer and Hay state (1995), "The entrance of the student into the teacher's realm of meaning is at the heart of most teaching." Further studies are necessary to determine whether the mathematics teaching by these non-mathematics teachers is effective, or whether more attention has to be paid to the training of prospective teachers who possess no experience in mathematics. There is, further, a question as to whether individuals who wish to enter the profession should be encouraged to have a broader university education than is currently the norm, including at least a few courses which contain mathematics in one form or another. This last is a political question, which can only be handled by all the faculties of education simultaneously.

One direct advantage of this study, then, is that student teachers can be made aware of the potential profile mismatches between teacher and students, and can learn to recognize and accommodate these differences. It may, further, be possible to introduce into the curriculum of both preservice and in-service courses, exercises which encourage the utilization of all the modes of mental representation, and hence which might assist teachers in developing their own abilities to use any mode in mental representation, if these can, indeed, be learned. The teachers who lack any exposure to mathematics in their post-secondary studies are of particular concern, since such teachers provide nearly all of the instruction in mathematics in the critical formative years of elementary school. Yet the potential for mismatches in profile between students and teachers appears particularly strong in the elementary grades, since many of the student-teachers in this study were not able to use particular modes of mental representation when asked to do so. Remedial courses for teachers should perhaps be highly practical, and closely tied to problem-solving.

Results from this study may also have implications concerning the mathematics curriculum at both the elementary and the secondary levels. Many of the students at the preservice level in the faculties of education possess a dislike for mathematics as a subject, a dislike developed in the elementary and secondary schools. This fact provides further support to the current drive for changes in the curriculum. The direction of change, however, should take into account two main points: first, the methods of teaching should provide more opportunities for the students to succeed, since so many of them do not succeed now; and second, the learning experiences of these students should encourage in them the utilizations of different forms of mental representation. Both factors might well be provided for at the same time, by offering more opportunities for hands-on manipulation, for problem-solving, for co-operative learning, for computer assisted instruction, and for concrete applications in mathematics courses, in contrast with the blackboard to pencil methods currently in use. This avenue of research appears promising, and has begun to be addressed. For example, a recent study of low-achieving grade nine students illustrated that use of computers in math classes can improve test results (Gurney, 1996). This is, however, only a beginning, and will require much more research before any firm conclusions can be reached.

This study has provided a starting point for an area of investigation on different rates of success in mathematics. It is clear from the data that students who possess no experience in mathematics do not have the same profile of use of the modes of mental representation as do those who have had experience. This is, however, only a starting point; more research is needed. In particular, this research does not address the question of whether the differences found are due to some deeper cause which, in itself, has encouraged students who prefer the visual mode of mental representation not to study mathematics in the first place.

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