# GRADE 9 MATHEMATICS IN THE COMPUTER LAB: AN INTRODUCTORY STUDY* 

PENELOPE J. GURNEY University of Ottawa


#### Abstract

A pilot project was conducted at a secondary school in which computers were used extensively in the teaching of mashematics at the grade 9 level. In this paper, the progress in mathematics of the students involved is compared to that of matched students in another s.hool who did not use computers. All student tests and exams were collectec during the school year from both groups, and analyzed. Gender appears to be a factor in the differences noted in results, but other factors, including attitude and type of software used, may also play a role. résumé. Un projet-pilote a été mené dans une école secondaire où les ordinateurs ont été utilisés pour la didactique des mathématiques en neuvième année. Dans cet article, je compare le progrès en mathématiques des étudiants qui utilisent régulièrement un ordinateur et de ceux dans un autre éccle qui n' en utilisent pas. Tous les tests et les examens des étudiants des deux groupes ont été conservés et analysés pendant l'année. L'un des résultats importants indique que le sexe peut être la différence principale, quoique d'autres facteurs, comme l'attitude et la sorte de logiciel utilisé, peuvent aussi jouer un rôle important.


The use of computers across the curriculum is becoming commonplace in many areas in the elementary schools, particularly in language arts. The secondary schools, however, and, more specifically, the mathematics classrooms of secondary schools, remain for the most part free of modern technology. This is a result of many factors, including the lack of software tied to the mathematics curriculum, the lack of sufficient computer equipment, and the difficulty that untrained, or inadequately trained, teachers have in using the existing systems effectively. The underlying reason appears to be that the teachers of mathematics are

[^0]not convinced that the use of computers will benefit mathematics learning and teachirg, and hence consider the purchase of hardware and software not to be a worthwhile endeavour (Capper, 1988).

There is some reseanch evidence supporting the use of computers in specific areas of the rnathematics curriculum. James Kaput (1992) provides a broad overview of current studies of such usage, and concludes that computers should have a place in the teaching of mathematics. Some authors (Fey, 1986; Thomas, 1991) have suggested that the use of computers in the nathematics classroom can improve the teaching and learning that takes place. The accounts are persuasive, but anecdotal, and do not mention supporting research. There have been a few studies (Heid, Sheets, \& Matras, 1990; Smart, 1987) which deal with one particular topic in mathematics such as algebra or geometry, and suggest that the computer might be helpful in teaching that one topic. Other authors (Dinkheller, Gaffney, \& Vockell, 1989; Farrell \& Farmer, 1988) simply assume that the use of computers in mathematics instruction is good, without providing evidence to support their statements. The arguments appear convincing that computers may make a difference in the learning of topics in geometry in particular, and perhaps in other areas of mathematics as well (McLeod, 1992). This does not answer the broader question, however, of cost effectiveness: is it worthwhile for a school or a school board to buy computers on a large scale, and have them used extensively in the teaching and learning of mathematics?

The present study addresses the issue of using computers in the mathematics class on a day-to-day basis. There are several possibilities inherent in the use of computers in the mathematics classroom. First, the computer may or may not be of benefit to the learning of mathematics in general by students. Second, the attitude of students towards mathematics may be affected, positively or negatively, by the use of computers. Third, there may be a cognitive element in the use by individual students' use of computers, in that the ability to use different mental representations may be different among those who use computers regularly in learning mathematics. Last, and perhaps of greatest importance to the school board, is the underlying idea that the computer may be of assistance in meeting the needs of groups of students with widely differing levels of ability, particularly those in grade 9 destreamed classes.

## THE NATURE OF THE STUDY

In the school year of 1993-4, a joint project was set up by the author and one school board in Ontario to examine the possibility of using computers in teaching mathematics. In one new school, a Macintosh labora-
tory was set up for use by the grade 9 destreamed mathematics classes, to permit two non-immersion grade 9 math classes to be taught, wherever possible, in the computer lab. The computer was to be used as a tool for review, for enrichment, and to assist in teaching topics. In other words, the computer was to be integrated into the course wherever its use was warranted in the mathematics curriculum. The expectation was that if the use of computer software could possibly help in the teaching of mathematics, any benefits would be more apparent and measurable in classes which contain the wide range of abilities found in destreamed grade 9 .

Of primary interest to both the teachers in that school and the school board, then, was the question of whether the students would benefit from this type of teaching. This benefit might take the form of academic advantage; it could involve greater liking for the subject; it might develop a greater ability in using different forms of mental representation. It was decided, moreover, that anecdotal evidence would not be sufficient to justify continuing with such a program: sufficient data would have to be collected to ensure that there was a measurable and meaningful relationship between good mathematics learning and computer use.

It was proposed, therefore, that the academic, attitudinal, and developmental levels of students at this one secondary school be measured, and compared to the like levels of similar students at a school where no computers are available for mathematics classes. By this means, not only would the staff of the pilot school know whether or not to continue such a program in future years, but the school board would find out whether this program could be of assistance in other schools.
Following discussion with board personnel, a control school was selected, in which the students of grade 9 mathematics would have no access to computers. This school was selected on the basis of similarity to the pilot school on the following factors: a) Both schools operate on a full-year system, in which mathematics is taught from September to June; b) In both schools, the French immersion students (often the most gifted) do not study mathematics with the non-immersion students, hence the non-immersion classes involved in this study contain a fairly high proportion of low achievers; and c) The socio-economic backgrounds of the students are similar.

The control group could not be selected from within the pilot school, because all teachers in that school made some use of the computer lab within the mathematics program, and hence the requirement that the control group not use computers would be a perceived hardship to the
teachers and students of this group. Furthermore, since the mathematics software on the machines in the laboratory was freely available to all students at lunch and after school, there seemed to be no way to ensure that students not in any pilot group would not have any access to the software.

## SUBJECTS

There are several grade 9 classes at the pilot school. Two were selected to take part in the stu:dy, so that those students who, with their parents, decided that they did not wish to take part, could easily be moved to other classes at the start of the school year. As it turned out, this did not occur, since the students involved in the group, and their parents, were anxious to keep any possible advantage, and would have objected strenuously to any attempt to remove a child from the study.
The complete set of subjects is described in Table 1. The 38 students in the pilot group were, in general, very weak in mathematics. In the mathematics exam given by the school board at the end of grade 8 , for example, only three students had a grade higher than $73 \%$, and 26 of them had a mark les; than $50 \%$.

TABLE I. Subject description

| Group | Malas | Females | Total |
| :--- | :--- | :--- | :--- |
| Pilot | 22 | 16 | 38 |
| Control | 26 | 31 | 57 |

The control school had three classes of non-immersion grade 9 students, all taught by the same teacher; from these three classes, 57 students (including 26 boys and 31 girls), with permission from their parents, volunteered to be part of this study. Since these students were not singled out in any way, but were taught in the traditional way within the standard classes, as part of the classes, the membership or non-membership of any student or students in the study did not present a problem to the teacher. The greater number of subjects in the control group, as compared to the experimental group, moreover, permitted more one-to-one matches of ability to be made between the two groups. There were both weak students and strong students in this control group: on the grade 8 mathematics test written by all grade 8 students in the board, 15 students received a mark greater than $73 \%$, while 21 had a grade lower than $50 \%$.

Since the complete set of subjects was not evenly matched in various factors, subsamples were formed from the pilot and control groups, all matched as to gender and on the results that they had achieved on eleven different topics tested in the common grade 8 mathematics examination of the previous year. There were 26 matches, consisting of eleven boys and 15 girls from each school. The other students either did not have matches, or had received extra help in mathematics from resource teachers during the school year, and hence could not be considered to have the same education as the average student in the class.

## PROCEDURE

The students in the control group were taught throughout the year in a standard, strongly disciplined way. The classes и ere highly structured, and earlier topics were constantly reviewed, in an effort to encourage mastery of the mathematics as a whole; in addition, tests were cumulative, again to encourage learning. No access to computers within the mathematics courses was, or could be, provided for these students.

During the first half of the year, the pilot group spent much of their mathematics class time in the computer lab: the mathematics lessons were delivered right in the lab, and software vas used both in the lessons themselves and in reinforcement of lessons. The software used came primarily from the Wings series of math teaching software, but also included graphing software and review tutorials. The students were encouraged to explore the tutorials after their lessons had been completed, so as to review elementary mathematics concepts on their own, in whichever order they wished. The lab was also available outside of class to any student in the school.

As the first half of the school year progressed, the two teachers in the pilot school continued to work ahead in integrating computer software into the mathematics curriculum. They were making an effort to teach all possible units using the computer, giving classroom instruction in units for which suitable software did not exist. At the completion of approximately sixty percent of the material in the course, the remaining topics in mathematics could not be effectively taught using the computer, so the classes were moved into regular classrooms, and taught using more traditional methods. The computers were still available to the students at lunch and after school, but were no longer employed as a part of, and during, the math class itself. Hence the results described below are in two parts: first, the results from the first part of the year,
when the experimental group was using computers, and then for the latter part of the school year, when the experimental approach had ceased.

## MEASUREMENT TOOI.S

## Regular testing and examinations

All students in both the pilot and the control group experienced the normal end-of-topic tests which measure achievement, done in every class at the end of a topic of study. This was carried out at regular intervals throughout the school year, and copies were made of all tests, for later inspection. When both sets of students had studied a specific topic, then the results of similar questions on the topic were compared and recorded. It would have been disruptive to the two sets of teachers to have expected them to have identical tests, but the questions asked were very similar, and hence direct comparisons were possible for the core topics and most of the optional ones in grade 9. The students also wrote mid-term and final exams, and a posttest based on the same kind of test items which had appeared on the original grade 9 entrance exam which they had written at the end of grade 8 . All of these were photocopied for schcol-to-school comparison.

## Attitudinal questionr:aire

One aspect of mathematics which is currently of interest is the student attitude towards it. There are many students who dislike mathematics, and drop out of mathematics study as soon as it is permissible to do so (Lafortune \& Kayler, 1992; Steen, 1990). Since all students in grade 9 must study mathematics, however, it is inevitable that the classes contain some students who dislike the subject. It is also possible that the use of computers in teaching mathematics will change the attitudes of some children. For these reasons, permission was asked from both the students and their parents in both groups, to have the students fill in an attitudinal questionnaire. The students filled in this questionnaire twice - once at the beginning of the course and once at the end. The attitude scale selected was that of Fennema and Sherman (1976), which was developed in a study of 1233 secondary school students, and has since become a standard instrument. The Fennema and Sherman scale is a Likert scale, with values ranging from "strongly agree" to "strongly disagree". Items included "Generally I have felt secure about attempting math" and "Most subjects I can handle, but I have a knack for flubbing up math". The scale was used, for example, in the evalua-
tion of a new college course (Hollowell \& Duch, 1991), where the experimental group consisted of 108 students, and the control group held 875 . Hence it was decided to use this attitude scale in the current study.

## Task interviews to examine mental representation

Are the students able to use visual, verbal, and enactive mental representations? It is thought by some (Aylwin, 1985) that the use of different forms of mental representation is developmental in nature, and that the amount of use of each of the forms of mental representation varies amongst individuals. It is known that there is a relationship between experience in mathematics and the ability to utilize different modes of mental representation in adults (Gurney, 1992); but little is known about the ability of children to use different modes of mental representation. There are really two questions at play here. First, do the students use all three modes of mental representation? Second, do the students use the most appropriate modes of mental representation when solving different problems in mathematics? Since a particular mathematical idea can often be represented in more than one way, the related ability to change representations, when necessary, is also important (Hiebert \& Carpenter, 1992).

It was felt that the question of which mode of mental representation is used by an individual student in the solution of a problem, could best be answered by individual sessions with student and researcher, in which the student was observed while attempting to solve problems. Six boys and six girls of different levels of demonstrated mathematical ability from each of the two groups (pilot and control), selected by their teachers, were therefore asked to solve problems. The problem-solving was done on a one-to-one basis, with each student explaining to an observer the methods of solution being used, and the reasons for following these methods; materials were provided so that each problem could be attempted using various forms of mental representation. The prob-lem-solving was carried out near the end of the school year, and required one session per student of up to 30 minutes. The student was audio-recorded as he or she described each thought process during the problem solution. The observer also made notes during the process, describing the actions taken by the student. The solution itself, any rough notes made by the student, the observer's comments, and the audio tape were analyzed afterwards to see how the student conceptualized the problem, the mental representation used in the solution, and the techniques used in the solution.

## RESULTS

## Attitude

The attitude of the students on entering the program was similar in both groups of students, in that those who were successful in the grade 9 entrance exam were more positive towards mathematics than were those students who were not successful. In both groups, however, the attitudinal scale gave mixed messages: students with consistently low grades (lower than $30 \%$ in a few cases) in mathematics stated that they could learn mathematics with no difficulty; yet some of the students with very high grades stated that they did not like mathematics, and were not ready for advanced work in mathematics; and some stated that they particularly enjoyed problem-solving, while at the same time they insisted that they disliked word problems. As expected, many of the students with poor grades did not see that mathematics could possibly have any relevance in life and work after secondary school. These attitudes did not change during the school year for more than one or two students in either group, indicating that it is probable that attitudes are formed much earlier in a school career. In general, however, the attitude of higher-achieving students in both groups was more positive than that of weaker students.

## Mid-term exam

One comparison examined in this study is that of the January mid-term examination results. It should be noted, however, that some of the early topics studied in mathematics by the two groups were different, so there may be little relevance for these interim results. In both the pilot group and the control group, however, the marks received on the mid-term examination were considerably higher than those on the grade 8 final mathematics exam. Table 2 illustrates the change for the 26 matched pairs.

There is no way to te $!$, from these data, whether the marks were higher on the midterm exams because of mark inflation (since many of the highest achieving grade 8 students were in immersion classes and hence were not part of this group), or because the students knew they were in a research project, or for any other reason. There is, furthermore, no way to tell whether the midterm exams in the two schools were of comparable difficulty, since the topics taught in the two schools in the first term were not identical. It is interesting, however, to note which students were successful, and which were not. It is considered normal that marks in grade 9 improve somewhat over those of the grade 8

TABLE 2. Comparison of scores of pilot and control groups on exams

| Group | N | Grade 8 <br> exam | SD | Grade 9 <br> mid-term | S D |
| :--- | :--- | :--- | :---: | :--- | :--- |
| Pilot | 26 | $52.65 \%$ | 12.4 | $67.3 \%$ | 16.3 |
| Control | 26 | 53.4 | 15.0 | 63.7 | 17.1 |

mathematics exit exam, but the expectation is for an increase of perhaps five percentage points. The main, and important, difference found on the January mid-term exam involved gender. The girls, in general, improved considerably in both schools. As can be seen in Table 3, this difference is statistically significant.

TABLE 3. The relation between January exam scores and gender - Analysis of variance

| Source | DF | SS | MS | F | P |
| :--- | ---: | ---: | ---: | :--- | ---: |
| Male between | 1 | 2442 | 2442 |  | $9.62^{*}$ |
| Female within | 50 | 12691 | 254 | . |  |
| TOTAL | 51 | 15132 |  |  |  |

A closer analysis of this difference in scores shows that there is a greater difference in scores at the pilot school than at the control school, in that this difference is statistically significant at the $99 \%$ level, whereas it is not so in the control group. This is illustrated in Table 4.

TABLE 4. Scores of pilot and control groups on exams by gender

| Pilot | Number | Grade 8 | Grade 9 mid-term |
| :--- | :--- | :--- | :--- |
| Girls | 15 | $53.1 \%$ | $73.87 \%$ |
| Boys | 11 | 53.4 | $58.36 \%$ |
| Control | Number | Grade 8 | Grade 9 mid-term |
| Girls | 15 | $55.1 \%$ | $68.09 \%$ |
| Boys | 11 | 51.4 | $55.9 \%$ |

In the pilot group, every girl in the class improved. Two-thirds of the girls had a mid-year grade which was more than 20 percentage points higher than the grade 8 score; the others all had grades which were from 10 to 19 percentage points higher. The seven girls who had an A ( $80 \%$ or more) at mid-year had received scores ranging from $38 \%$ to $68 \%$ on the grade 8 exit exam. A description of these scores is set out in Table 4 above. The boys showed some improvement also, but not to as marked a degree. In particular, one-quarter of the boys had a mid-year grade
which was more than $20 \%$ higher than that of the grade 8 exam, and $19 \%$ had grades which were from 10 to $19 \%$ higher. The four boys who had an A at mid-year included the only three in the class who had an A on the grade 8 exit exam, plus one who had received only $47 \%$ on the grade 8 exam.

In the control group there was also some improvement, and again the improvement was more striking for the girls than for the boys. Approximately $12 \%$ of the boys and $44 \%$ of the girls had a score at mid-year which exceeded their grade 8 exam score by 20 percentage points or more, while an additional one-fourth of both boys and girls surpassed their grade 8 scores by 10 to 19 percentage points. The number of A grades at mid-year was a little higher, with the four boys and six girls increasing to six boys and ten girls. Two boys and one girl went down from an A to a lower grade, while three boys and five girls went from Bs and Cs to the A grade.

In neither the pilot group nor the control group was anything known about their instruction in grade 8 mathematics before they entered secondary school. The only thing known was that all grade 8 teachers of mathematics follow the same curriculum, and that all students write the same mathematics exam at the end of the school year. The order of instruction was therefore an unknown variable in the experiment. A second unknown effect was that of destreaming. This was the first year in which students in grade 9 had not been placed in different classes according to demonstrated ability in subject areas such as mathematics. As a result, some students followed a more difficult curriculum than they would have followed in previous years, while others had followed a less challenging pregram. At the midway point, this did not appear to have harmed the students.

In examining the individual topics taught during the year, topic by topic, some interesting trends appeared. In the control group, there were several streams of students. There were, first of all, those who did consistently well on each selected problem, and those who did consistently poorly. These students neither improved nor changed for the worse during the course. Next, there were students who started off well, and became consistently worse during the year. There were also those who improved steadily during the year. Finally, there were those who alternated between good and poor work.
This did not occur in the pilot group. In this group, there appeared to be only two streams. The first consisted of those who did consistently
well, while the second alternated between good and poor work. Hence, it appeared that the students in the pilot group had taken control of their own progress, and worked to improve it. The streams which were evident in the control group data did not appear in the data from the pilot group.

## END-OF-YEAR RESULTS

The end-of-year results for the pilot group did not closely resemble those of the mid-year. They were not as encouraging, for either the group as a whole or for the girls. Two different measures were examined at the end of the school year. The first of these was simply the final exams in the two schools, as compared to the grade 9 entrance exam and the January exam. This appears in Table 5 below.

TABLE 5. Exam scores for two samples by gender

| Pilot | Number | Grade 8 | Grade 9 <br> mid-term | Grade 9 <br> final |
| :--- | :--- | :--- | :--- | :--- |
| Girls | 15 | $53.1 \%$ | $73.6 \%$ | $66.2 \%$ |
| Boys | 11 | $52.0 \%$ | $58.36 \%$ | $57.7 \%$ |
| Control | Number | Grade 8 | Grade 9 | Grade 9 |
|  |  |  | mid-term | final |
| Girls | 15 | $55.1 \%$ | $68.09 \%$ | $58.9 \%$ |
| Boys | 11 | $51.4 \%$ | $55.9 \%$ | $52.36 \%$ |

As can be noted in Table 5, the improvements found on the grade 9 mid-term did not persist for the female students to the end of the year, although the girls in the pilot sample apparently did retain more of their gains than did either the boys or the control sample.

The posttest scores could not be used in analyzing results, since one of the classes in the pilot school did not write the test under test conditions, but rather used it as an optional review in preparation for the grade 9 examination. Copies of these final examinations had been made, however, and the examinations for grade 9 mathematics in both schools contained questions similar to those of the grade 8 exit exam for more than $90 \%$ of the topics. This fact is due to the nature of the destreamed grade 9 curriculum, in which an attempt is made to bring all students to a common level. Hence these questions were used in the comparison of the grade 8 and grade 9 achievemert levels in mathematics topics. The results appear in Table 6.

TABLE 6. Scores on common elements: Beginning and end of grade nine

| Pilot | Number | \# pass | pretest | \# pass | posttest |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Girls | 15 | 10 | $52.1 \%$ | 14 | $62.73 \%$ |
| Boys | 11 | 7 | $53.6 \%$ | 7 | $62.91 \%$ |
| Control | Number | \# pass | pretest | \# pass | posttest |
| Girls | 15 | 10 | $56.2 \%$ | 11 | $62.67 \%$ |
| Boys | 11 | 7 | $52.4 \%$ | 7 | $51.18 \%$ |

From this table, it can be seen that one group of students, namely, the boys in the control group, did not improve their scores on basic mathematical questions during the grade 9 year. The general difference in scores was not statistically significant, due probably to the small sample size; yet the nature of the individual scores of these boys is interesting. Of the six boys who had failed the grade 8 entrance exam, five failed the equivalent questions at the end of grade 9 , and four of these had scores as much as $30 \%$ lower than in grade 8 . This occurred despite the considerable amount of extra help provided for these students during the course of the year. These grades kept the average at the grade 8 level despite the improvement of some boys during the year. Overall, in the control group, eight students failed the grade 8 material on the grade 9 exam, while two of the pilot group did. Hence overall, the girls from both groups improved their scores on the fundamental grade 8 topics repeated in grade 9 , as did the boys in the pilot group.

## Individual interviews

One-on-one intervitws with students in the control group and the experimental group were conducted near the end of the term. The teachers were directed to select four students who were good in math, four who were poor in math, and four students who were average in mathematics; they were further asked to ensure that half of each set were boys, and half girls. Hence the selection was not entirely randomized, in order to ensure that each sample held both girls and boys of various demonstrated levels of ability. During each interview, the student had four problems to solve, while providing a running commentary on his or her thoughts during each attempt to solve a problem. The problems were of different kinds: each student had one problem which could be solved only by drawing a diagram; one which could be solved by step-by-step logical argument; and two which could be solved in different ways. Many of these were chosen from the Ontario Assessment Instrument Pool for grades 7 to 9 . During each interview, the
student sat at a table with paper of various sizes, graph paper, protractors, rulers, compasses, and all kinds of manipulative material within easy reach.

It was obvious in the interviews that the students were not used to the kinds of problems with which they were presented, in that they really had no idea of what to do with the problems presented. The ways in which the pilot and control students dealt differently with the problems are, however, instructive. First, and perhaps most important, is that very few students drew any diagrams at all, let alone for those questions which could not be solved without one; and those few who did draw diagrams, often did not draw correct diagrams which might help them in the problem solution. There were two individuals, however, one boy in the control group and one girl in the pilot group, who did draw appropriate diagrams where such diagrams would help; there was also one boy who used the manipulative material on the table to build a scale model to solve a problem in which such an approach was very useful. The boy in question, from the pilot group, was not considered by his teachers to be a good student of mathematics, but he was the only student to solve the problem correctly.

There were differences between the two groups in some aspects of problem solving, but these differences did not have anything to do with the use of computers in the classroom. The students in the control group had obviously been taught some basic problem-solving strategies; most of them wrote down the given information on a separate sheet of paper, and then looked for something which could be answered. If the problem mentioned percentages, for example, then percentages were determined in a manner which was mathematically correct, though not all the percentages found were those asked for. The students in the pilot group had no such strategy in mind; these students, for the most part, decided on a plan of action by guesswork, or attempted, unsuccessfully, to obtain hints from the observers. They showed no evidence of knowing how to solve problems, or even of having met any of the common strategies in the grade 7, 8, or 9 Guidelines For Mathematics (1985). Furthermore, these students did not understand sufficient mathematics to begin to solve some of the problems; for example, the majority of these students did not know how to calculate a percentage, how to find an area, or how to substitute values for variables in an equation. In fact, six of the 12 students in the pilot sample simply refused to attempt problems which they did not feel confident about solving, compared to none of the control sample. Specifically, a problem which had several
parts, or which required a diagram to be drawn, was more likely to be refused, whereas a short, one-sentence question would most likely be attempted.

The questions dealing with logical thinking were well done by the majority of students in both groups; all students demonstrated an ability to think clearly and reason mentally to come up with a conclusion which made sense. The other problems, however, did not fare as well: the standard problems, and the problems requiring diagrams, were not, in general, well-done. There was little evidence of ability to visualize situations, or to relate the questions to mathematical concepts. A problem which had to be broken into several small problems was, in general, poorly done, since the students who attempted this kind of question often ended up getting lost in the detail. The control sample did, however, succeed in solving approximately two-thirds of each problem of this type, while those in the pilot sample managed at most one-quarter of the problem.

There was a noticeable difference in both samples between the behaviours of boys and of girls, in that the boys attempted to do everything mentally, while the girls would write things down. This was particularly noticeable in the control sample, where all students tried to solve all the questions, but also existed in the pilot sample, for those questions which were attempted. Only after all mental efforts had failed, would the boys write anything down. As a result, the arithmetic computations done by the boys were often incorrect, even when the methods were correct.

When the diagrams which had been drawn were examined, they bore little resemblance to the problems which they purported to represent; the students were not able to visualize the problems. For example, one of the questions dealt with a tree which had a rope attached to the top of the tree and the ground, while another talked of a cliff, and an observer who walked directly away from the cliff. For neither problem was a good diagram drawn: in the tree problem, not one student drew a line representing the rope; while in the cliff problem, the cliff was not drawn as a vertical line (to the ground), but rather was represented by $>$, neither horizontai nor vertical, and with no connections to anything. Measurements which were provided served only to verify one of the common myths of problem-solving, namely, that "if a problem contains two or more numeric values, then the solution can easily be found by combining these values to obtain a numeric result." Four students (three from the pilot sample and one from the control sample) combined angle mea;ure and side measure to obtain either a sum or a
product, which somehow seemed reasonable to them, rather than simply draw a diagram and measure the desired value. Only two students out of the 24 , one from each group, tried an approach which had a chance of leading to the correct solution.

At the end of each problem-solving episode by students of the pilot sample, a set of six questions were asked about the use of computers in their mathematics classroom.

These questions asked the student to specify their usage of the computers both inside and outside of the mathematic; class, the degree of assistance provided by the computer software in learning mathematics, and whether the students enjoyed the process.

One of the students had replies which were different from the others in the sample; this one student did not use the compiters much during the math class, and did not like them, while the other eleven stated that they had used the computer extensively, and did like using this technology. The students stated that the computer was "fun", and that using the computer was easier because "you don't have to write everything out on paper". They felt, furthermore, that the computer is better than ordinary classroom instruction because the computer is an individual assistant, who can provide immediate help and immediate correction. Of the twelve students, however, five felt that they could have learned just as much from a textbook, or from normal clas; interaction, but four out of the five did enjoy the computers more than regular class. None of the students used the computers outside of class for mathematics, although they did use computers for other things, such as word processing.

In the pilot classroom, the software used was found to be inadequate for direct instruction, and was used more for practice and drill. The teaching method soon settled into a short lesson, followe -1 by a brief worksheet, then finally the computer component. The topics dealt with by the software included calculations, order of operations, integer arithmetic, fractions, exponents, algebraic relations, and word problems, while utility software, such as function graphing, charts, and spreadsheets, were also used.

## DISCUSSION

It is clear from the attitudinal questionnaire that many of the students in grade 9 have an unrealistic low opinion of the benefits of mathematics, in that many of them do not believe that mathematics is necessary for the world of work. Furthermore, these students do not know whether
they have any skill in this area, nor what their strengths and weaknesses are. One particular student, for example, who stated that mathematics is easy, did not have a mark on any test greater than $40 \%$, while an excellent student, with excellent marks, thought that she could not do advanced mathematics. It may be optimistic, therefore, to think that one can change the attitudes of such students towards mathematics at this late date in their schooling.

The results from the examinations on content, of both grade 8 and grade 9 material, show that in the pilot group, grade 8 basic material was learned during the grade 9 year, and that the new grade 9 topics were better completed by the girls than the boys. This is illustrated by the fact, as shown in Table 4 and Table 5, that the scores received on the grade 9 final ( topics from both grade 8 and grade 9) were higher for the girls, and lower for the boys, than the score on the grade 8 material alone. For the control group, the girls appeared to have learned more of the grade 8 math topics and fewer of the grade 9 topics, while the boys did the opposite.

It is particularly interesting to note that the girls in both the pilot group and the control group showed greater gains in mathematics achievement than did the boys during the first part of the year. In the control group, the higher achievement continued while elementary material was retaught, while in the pilot group, the improvement coincided with the use of computers in the classes. It is possible that the girls in the control group appreciated the mastery learning approach to mathematics, and the second chance to learn elementary topics which had not been mastered earlier. In the pilot group, the achievement levels of the girls improved significantly more than the boys while the computers were being used, but these gains were not all maintained during the non-computer portion of the course. It is possible that the use of computers provided these girls with individual attention which could not exist in a normal classroom; it is also possible that the novelty of the situation provided the girls with greater incentive to work at mathematics, since few girls had used computers previously other than for word processing, while most of the boys were familiar with computer games.

At this time, it cannot be concluded that the use of the computer software provided in the pilot mathematics classroom was of significant benefit to the students studying new material in mathematics. There may well have been an advantage for both boys and girls in the learning of the grade 8 material which they had not learned previously, as shown
in Table 5; but in learning new material, the results are not clear-cut. What is known, however, is that much of the software available during the study did not match precisely the mathematics curriculum being taught, and hence that better, and very much more expensive, software would have to be written or purchased in order to test adequately the usefulness of the new technology. The software available in the market does not, in general, match the curriculum being taught in our schools, and appears suited only for review of previously learned material. This review material does have potential for use by students who need more reinforcement than is feasible in a regular classroom. Of particular importance are those programs which do more than provide "yes or no" feedback, but rather lead the student through a solution, step by step: The teacher still has to provide other methods of solution, but the computer can provide one-to-one attention where needed.

At this point in the research, more questions have been raised than have been answered. The question of effectiveness of computer software has not yet been adequately answered, since this would require more expensive software than is currently reascnable. The currently available software, however, will not replace any instruction by teachers. Furthermore, it is indeed possible that girls who have not mastered the basics in mathematics can benefit from standard software, but this is only suggested from the data, not proven beyond doubt. In addition, the gender differences found in the results in both grade 9 groups suggest that possibly new approaches must be found for adolescents who have not reached expected levels of achievement in mathematics. As shown in Table 5, students in the pilot (computer) group did learn enough of the earlier mathematics topics to warrant a pass on these topics, but the average scores were still only at a C level, which is not very high for such material. The girls in the control group, but not the boys, also improved their scores only to this level, which is disappointing in a mastery approach.

On another general topic, the whole issue of the teaching and learning of problem-solving needs to be addressed. The difficulties in this domain appeared to be at a deep level: even those students who had memorized a list of techniques for problem solution seemed unable to apply them appropriately. Moreover, there was a lack of understanding of the actual questions being asked, and a lack of ability to visualize the situations in which the questions were posed.

It is important to consider this study as a starting place for future study, on questions not only of teaching methods, but also of attitudes towards
mathematics, preconceptions of students towards mathematics, and gender differences in the study of mathematics.

In conclusion, a board of education cannot entrust the mathematics education of students to computer-based materials unless it is prepared to make a very significant expenditure for superior software. Since such superior software does not yet exist for complete courses, or even for complete units, the question is academic at this time. It was originally thought that computer software might replace the need for textbooks, but this has not hapr ened either: textbooks are still a necessary part of the mathematics program. In order that current software be used effectively, the classroom teacher must teach the topic to the student in the same way as the computer software, which runs counter to the current philosophy of multiple approaches leading to understanding. The teacher must also teach the student how to use the software, since there is little standardization in the user interfaces for different pieces of software. Furthermore, much of the inexpensive software addresses only one-line questions, and hence is not suitable for advanced work. The teacher ends up having to do more work in integrating computers into the curriculum than the integration is worth.

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PENELOPE J. GURNEY is an assistant professor in the Faculty of Education, University of Ottawa, specializing in mathematics and computer science methods courses, and methods and interpretation in quantitative research.
PENELOPE J. GURNEY est professeure adjointe à la Faculté de l'Education de l'Université d'Ottawa et se spécialise dans la didactique des mathématiques et de l'informatique, et les méthodes et interprétation en recherche de type quantitatif.


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