Nineteenth-Century Canadian School Mathematics

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***Quantity** is any thing that can be increased, diminished, or measured.* **Mathematics** *is the science of quantity.*

— *1858 definition.*[1]

The present generation of Canadian educators is sufficiently removed in time from the schools and programs of the nineteenth century that Victorian practices in curriculum and instruction are tending to take on attributes of the legendary. However, abundant evidence can be found of the actual course contents and approaches in most, if not all, nineteenth-century subject areas. This is particularly true in school mathematics.

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The mathematics of the Victorian classroom came to comprise arithmetic and mensuration, readily justified as of "distinct value in the business of life,"[2] plus algebra and Euclid, which had entered the school from the college program.[3] (Even in the 1880's, university undergraduate mathematics offerings consisted mainly of calculus and analytic geometry.[4]) "Up to the end of the last century," an English study has noted, "school mathematics . . . generally consisted of arithmetic, algebra and geometry (Euclid) and there was very little correlation between the subjects."[5]

The dominant role that the textbook came to assume in the Victorian mathematics class cannot be doubted, and the single-volume Arithmetics, Algebras, and Euclids, accordingly, are of great interest. An earlier period, however, had known few textbooks, and a small book of rules and examples of arithmetic, *The Tutor's Assistant,* published in Montreal in 1845, offers insight into classroom procedures.[6] The author, one Francis Walkingame, indicates that an accepted practice in the mathematics class was for the teacher to write out "the rules and questions" in each pupil's notebook, and he says:

> When a master takes upon him that laborious, (though unnecessary,) method of writing out the rules and questions in the children's books, he must either be toiling and slaving himself after the fatigue of the school is over, to get ready the books for the next day, or else must lose that time which would be much better spent in instructing and opening the minds of his pupils.

Walkingame, accordingly, had prepared handwritten booklets, one for each level of instruction, and had encountered a new difficulty.

> Where there are several boys in a class, some one or other must wait till the boy who first has the book, finishes the writing out of those rules or questions he wants, which detains the others from making that progress they otherwise might.

Walkingame, therefore, ventured to publish his "compendium."

A number of Canadian Arithmetics did exist in Walkingame's time, and many others were to follow, Canadian products as well as imports from England and the United States. Indeed, fifteen years later, John Bruce, the Quebec school inspector, was moved to write about the teacher who, because of his own limitations, "is obliged to follow the very letter of the book."[7]

Bruce, in an early number of *The Journal of Education for Lower Canada,* spoke of the teacher "who knows nothing of the subject but what is contained in the book before him, and who knows *that* only as he reads it during the intervals occasioned by the hesitations of the class." Observed Bruce, "The tendency of such teaching is to discourage thought."

Laying claim to being the first "Canadian Arithmetic in the English language," *Adams' New Arithmetic,* by Daniel Adams, a physician, was published at Stanstead in 1833 and at Sherbrooke in 1849.[8] It is antedated by a quarter century, however, by a French-language text, *Traité d'arithmétique pour l'usage des écoles,* printed in Quebec City in 1809.[9] Both books are startlingly comprehensive, extending from the rudiments of "cyphering" to cube root, the progressions, and the more sophisticated applications to banking and commerce. Thus, the 1809 treatise asks:

> Un homme, partant pour voyage, fit 10 lieues la pre-

> mière journée et se rendit de 8 jours, augmentant sa marche de 5 lieues par jour. Combien fit-il la dernière journée?

Adams, as late as 1849, presents three monetary systems: the "currency of the country," Halifax currency, denominated in pounds, shillings, and pence; "old currency" of the French regime, in livres, sous, and deniers; and "Federal money" of the United States, in eagles, dollars, dimes, cents, and mills. Adams asserts: "The importance of the *principal* and *essential* alteration of this book, *viz.*, the adaptation of it to the currency of the country, will not fail to be observed by every one."

An antiquated system of "French square measure," of which vestiges persist in Quebec to this day, is tabulated, through the square French foot, *toise*, rod, *arpent*, and league. An English duodecimal concept is interestingly extended, in linear and surface measurement.

> Duodecimals are fractions of a foot. . . . A foot, instead of being divided into ten equal parts, is divided into twelve equal parts, called inches, or *primes*, marked thus, ('). Again each of these parts is conceived to be divided into twelve other equal parts, called seconds ("). In like manner, each second is conceived to be divided into twelve equal parts, called *thirds* ("'); each third into twelve equal parts called *fourths* (""), and so on to any extent.

Adams points out, accordingly, that a *fifth* is 1/248832 of a foot, and extends the notation to area by giving as the product of 371 feet 2′6″ multiplied by 181 feet 1′9″, the duodecimal 67242 feet 10′1″ 4‴ 6⁗.

The contents of *Butler's Arithmetical Tables*, "designed chiefly for the Use of Young Ladies" and published in Montreal in 1855, include a surprising diversity of items.[10] Among them are "current coin," Roman and Greek coin and moneys, Kings and Queens since the conquest, wage rates, "Old Ale and Beer Measure," populations, and "astronomical characters." In the tables as in the school arithmetics, the attempt is to provide or predict the useful. That this is not easy is borne out by Thorndike's observation that "percents were once as rare in the shop and on the street as fractional exponents are now."[11]

John Herbert Sangster, a Toronto physician, undertook "at the suggestion of the Chief Superintendent of Education for Upper Canada" to produce "a *complete* text-book on the subject of Arithmetic," and his *National Arithmetic in Theory and Practice,* issued in 1859, enjoyed several decades of Canadian use.[12] "Urging on his fellow-teachers" with recommendations on the need for drill, memorization, revision, absolute accuracy, care with operational signs, and originality in problem preparation, he seems decades in advance of his time. "Designed for the use of Canadian schools," the National Series of texts found widespread endorsement when, in the words of a Quebec school board report, books in use were "very various."

The popularity in the Canadian provinces of Hamblin Smith's *A Treatise on Arithmetic,* the English classic, led W. J. Gage and Company of Toronto to bring out a Canadian edition in 1877.[13] The Canadian revision was by Thomas Kirkland and William Scott, respectively principal and vice-principal of the Normal School, Toronto. The same authors produced the following year, "intended as an introductory text-book to *Hamblin Smith's Arithmetic,*" their *Elementary Arithmetic on the Unitary System,* with editions in 1878, 1888, and 1895.[14] The second edition claimed authorization in Ontario, Quebec, Nova Scotia, Prince Edward Island, Manitoba, Northwest Territories, British Columbia, and Newfoundland.

The Hamblin Smith text is divided into two parts, Pure Arithmetic and Commercial Arithmetic. The former, in 113 pages, extends from, "Write in words the numbers expressed by the following figures: — 7, 13, 45, . . ." (p. 5) to such a demand as, "Find . . . the sixth roots of 24794911296; 282429.536481" (p. 113). Topics in "Commercial Arithmetic" include, predictably, English, Canadian, and United States currencies; measures of time, length, surface, solidity, capacity, and weight; fractional measures (e.g. 3/73 of a year + 9/56 of a week + 7/12 of an hour); decimal measures; simple interest; compound interest; present worth and discount; equation of payments; averages and percentages; profit and loss; stocks and shares; division into proportional parts; alligation; exchange; ratio and proportion; the metric system; measurement of area (e.g. carpeting floors, papering walls).

Kirkland and Scott claimed as a chief virtue of their book the new approach to the presentation of arithmetic topics. Previously, they note, a rule had been stated first, an example illustrating the rule followed, and the reason for it had come last, while "exactly the reverse of this is adopted by all good teachers." The examples and illustrations precede and lead up to the enunciation of the rule, whenever a rule is considered necessary.

The single-volume Arithmetics in general use through most of the nineteenth-century provide little insight into the assignments of the various grade levels. The first "Course of Study" for Quebec Protestant schools, released in 1883 or 1884, is of some interest in this regard.[15] Four elementary grades, three model school grades, and three academy grades, are provided for.

First Grade:	Counting. Mental Arithmetic. Addition and Subtraction of numbers of three figures. Reading and Writing Nos. to 1,000. Multiplication table to 6 times 9.
Second Grade:	Mental Arithmetic. Four Simple Rules to Long Division, inclusive. Multiplication Table. Avoirdupois Weight. Long and Liquid Measure.
Third Grade:	Mental Arithmetic. Long Division. Compound Rules. Simple Examples in Fractions. Dry, Time, Square and Cubic Measures.
Fourth Grade:	Mental Arithmetic. Fractions, Decimals, Elementary Interest and Percentage.

The Fourth Grade program included, as well, an introductory course in "Bookkeeping," requiring "single entry bookkeeping, making out accounts, receipts, orders, etc."

More advanced students, in the Model School or the first Academy year, met percentage applications in "Commission, Brokerage, Insurance, Interest and Profit and Loss." They concluded their program with "Present Worth, Discount, Equation of Payments, Stocks, Partnership, Square and Cube Roots."

Mensuration, the study of measurement, has been described as "one half of arithmetic," and in the late nineteenth century it may have had the stature of a separate subject. In *Practical Mensuration*, published in Owen Sound in 1893, the author, C. A. Fleming, argues:

> There is scarcely a transaction in every-day life to which [Mensuration] does not apply, from the pur-

> chase of a yard of calico to the great railway contracts that require the time of their execution to be counted in years.[16]

Devoting forty pages to applications of measurement, Fleming treats such "common measurements" as those relating to wood, lumber, land, lathing, plastering, painting and kalsomining, papering, stone and brick work, shingling, carpenter work, fencing, cisterns, timber measure, measures of grain and hay, gauging, and shoemaker's measure.

As in this representative example from Hamblin Smith, most mensuration problems were relatively straightforward, called for some care in computation, and were little concerned with "rounding off" or significant digits:

> How many flag-stones, each 5.76 ft. long and 4.15 ft. wide, are requisite for paving a cloister, which encloses a rectangular court 45.77 yd. long and 41.98 yd. wide, the cloister being 12.45 ft. wide?

Other late Victorian examples, less frequently encountered, proved anything but routine:

> If a rifleman can plant 11 per cent of his bullets within a circle of 1 foot in diameter, at the distance of 100 yards, find the diameter of that circular target which he might make an even venture to hit the first shot.[17]

The stated answer, to an improbable degree of precision: 2 feet 5.2656 inch.

Algebra, "a science which treats of the relations of numbers,"[18] was shown to be useful in circumstances where arithmetic methods were difficult to apply. Kelland, a Scottish clergyman, made this point in a textbook of 1869.

> In a great number of arithmetical operations, it is possible to carry on a continuous train of reasoning by means of which the various data are successively introduced in their proper places, and the conclusions to which they respectively lead, combined and worked into the final result. . . . We must of necessity have in view the conclusion itself, or something involving it, even at the very outset of the solution. . . . The science of Algebra has for its primary object the exhibition

> to the eye, of all the operations which in this case would have been represented only to the mind.[19]

"Manipulative skill seemed to be the aim of the teaching," it has been asserted, "and the skill demanded went far beyond that which the non-specialist was ever likely to use."[20] The Victorian educator was not unaware of this aspect of algebra teaching, and Hall and Knight cautioned, in 1885, "lest first lessons should degenerate into a more mechanical manipulation of symbols, uninteresting and uninstructive, because little understood."[21] Approach and content varied little from textbook to textbook, the operations with algebraic expressions, the solution of equations, the application to "type problems," and an unconnected final chapter on graphs. Apart from integration of the graphwork, little essential change in approach is found through 1850, so the format of a traditional algebra is not hard to recall. "Word problems," undeniably, have altered, so the question from Kelland's *Algebra,* can claim a certain antiquarian charm. So can this "clock problem," from the nineteenth-century *Algebra* of the Ontario authors, Robertson and Birchard.

> It is between 2 and 3 o'clock, but a person looking at his watch and mistaking the hour-hand for the minute-hand, fancies that the time of day is 55 minutes earlier than it really is. What is the true time?[22]

Problems dealing with such "seeming trivialities" are "to be welcomed rather than condemned," according to Carson, for they "exemplify the handling of rates in ways which are more real to young children than others which have more actual utility."[23]

Since Euclid is a work lending itself to citation by book and proposition, if not chapter and verse, nineteenth-century limits in geometry instruction are readily ascertained. The first three books of Euclid constituted the subject matter for a 150 mark matriculation paper to McGill University in 1875. Quebec limits of 1883, unaltered through 1898, reflected this requirement:

Academy, Grade I :	Definitions, Book I, 1-26;
Academy, Grade II :	Books I and II;
Academy, Grade III:	Books I, II and III.

Since the assignment of propositions is not very different from that of sixty years later, a look at examinations reveals a great, perhaps unexpected, difference in approach and emphasis.

The High School of Montreal, in a paper attributed to sessional examinations of 1861, asked for "I.11" by number, and required, "Give the propositions used in the construction and demonstration of I.48 Pythagorean converse."

Teachers aspiring to Quebec's Model School Diploma in 1882 had as 20 mark obligatory questions:

> Draw the figures of the 4th, 8th, and 24th propositions in Book I, the 11th in Book II, and the 9th in Book III.
>
> Give the general enunciation of the 5th proposition in each of the Books. Enunciate also the propositions you consider the most difficult in Books I and II.

Change came at the turn of the century. The 1900 edition of the *Course of Study* added to Academy, Grade I requirements, *at least five easy deductions.* Ten deductions were to be done in the second grade; fifteen in the third. In 1906 it was directed that the Academy Grade 1 paper have "at least one simple deduction among the compulsory questions." Minutes of the McGill University Matriculation Board for 1909 contain the telling entry:

> That candidates should be informed that it will be necessary to buy not only the text-book but also a set of drawing instruments.

Euclid, clearly, would never be the same! However, Mayor's observation of 1959, that "a resourceful teacher could use a translation of Euclid's *Elements,* written more than two thousand years ago, and with appropriate mimeographed exercises meet the course syllabus requirements," was essentially true in Quebec of 1900 as it is in most of Canada even in this decade.

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The prevalent conception of Victorian school mathematics as manipulation to near perfection, rote learning, and inherent irrelevance represents a distortion and an oversimplification. It does no justice, for example, to the very real efforts of the nineteenth-century author or schoolmaster to prepare his youthful charges for a harsh, demanding world. Bookkeeping in Fourth Grade! Arithmetic, certainly, shows attempts to drill the useful, even to predict the useful, a difficult feat during a time — not unlike ours — of unprecedented technological and social change. Strict accuracy was, of course, at a premium when business was governed by hand ledgers (the black and red ink pots!), before the typewriter, cash register, desk calculator and computer shifted the demand to different skills. It cannot be denied that algebra carried manipulation — "symbol shoving" — to unneeded lengths, asking little in terms of understanding, and geometry often failed to call for original thought; it is also evident that emphasis was on the "mental discipline" of the subject and the assumed carryover of learned patterns of logical thought. Nevertheless, nineteenth-century mathematics instruction should not be lightly dismissed and from an understanding of its strengths and weaknesses may emerge a fuller appreciation of the changes being recommended and implemented in today's curriculum.

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